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LUNAR PHYSICAL PARAMETERS STUDY

PARTIAL REPORT NO. 12

MEASUREMENTS RELATED TO THERMAL CONDUCTIVITY

WORK PERFORMED UNDER J.P.L. CONTRACT NO. N-33552

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RE-ORDER No. 61-422

MEASUREMENTS RELATED TO THERMAL CONDUCTIVITY

TABLE OF CONTENTS

1:794.38-0

RE-ORDER No. 61-422

MEASUREMENTS RELATED TO THERMAL CONDUCTIVITY

In support of development work done on a method for measuring the thermal diffusivity of the lunar surface, a mathematical study has been undertaken. This study was designed to verify the feasibility and design calculations previously reported[1], and to provide some experience in calculation methods which might be used in the interpretation of data actually obtained.

The apparatus built for breadboard tests was used as the basis for calculation. The apparatus consisted of a flat circular disc, 12 in. in diameter, which was shielded from direct rays of the sun by a reflector directly above it. This allowed the disc to assume a lower equilibrium temperature than if it were in direct sunlight. The entire bottom surface of the disc was covered with an electrical heating element cut in a spiral of archimedes configuration. The lower surface was blackened to make the emittance as high as practical. A 1 in. diameter hole through the center of the disc allowed a radiometer mounted between the disc and reflector to view the area of the surface directly beneath the disc.

The calculations done were based on experiments which would be performed in the following manner:

1. <u>An experiment performed during the lunar day</u> - The disc, at a temperature lower than that of the surface, is placed

[1] Lunar Physical Parameters Study, Partial Report No. 8, Design Calculations, Measurement of Thermal Diffusivity, Texaco Inc., May 8, 1961

near the surface and parallel to it. The temperature of the disc is maintained constant[2] and the temperature of the surface is measured by the radiometer at a number of times, up to perhaps an hour after placement of the disc.

2. An experiment performed during the lunar night - The disc is heated electrically and held at a constant uniform temperature. The disc is then positioned over the surface and the temperature measured by the radiometer at a number of times.

The actual calculations were approached in two phases. The first phase consisted of attempting to find reasonable sets of physical parameters which would allow calculation of temperatures and temperature gradients in the near-surface region. Results of such calculations could then be used as initial conditions for the calculation of the expected experimental results. These results also provide estimates of optimum times for performing the experiments, when the temperatures and temperature gradients are slowly varying with time over the surface area of interest. The second phase involved calculations of expected results of the experiments described earlier with various sets of assumed physical parameters.

Phase 1

The first phase calculations were based on a layered model.

[2] Its temperature actually rises slightly, but with large heat capacity of the disc this is minimized and was ignored in the calculation. Some additional advantage might be obtained by supplying a small amount of thermostatted power to the heating element sufficient to balance radiation losses from the disc which can be cut off when the disc is positioned.

The surface was assumed to be flat and of uniform character. The medium was treated as a semi-infinite slab made up of a finite number of parallel homogeneous, isotropic layers. Thus, the heat equation in the interior of any layer is;

$$\frac{\partial v(x,t)}{\partial t} = \kappa \frac{\partial^2 v(x,t)}{\partial x^2} \qquad (1)$$

where,

κ is the thermal diffusivity of the layer

v is the temperature

t is the time, and

x is the depth variable.

At the surface, $v(o,t)$ is known from astronomical data and the net heat flux conducted into the surface is

$$f(o,t) = \alpha_v q - \epsilon_s \sigma [v(o,t)]^4 \qquad (2)$$

α_v = average reflectivity of surface in solar spectrum

q = incident solar radiation of the form $q_o \sin \Omega t$ during the day and zero during the night

q_o = solar constant

Ω = synodic frequency $\times 2\pi$

ϵ_s = emittance of the surface in the infrared

σ = Stefan-Boltzmann constant.

The heat equation rewritten in terms of the flux becomes,

$$\rho c \frac{\partial v(x,t)}{\partial t} = - \frac{\partial f(x,t)}{\partial x} \tag{3a}$$

$$f(x,t) = -K \frac{\partial v(x,t)}{\partial x} \tag{3b}$$

ρ = density of medium

c = specific heat

K = thermal conductivity = $\kappa\rho c$.

A closed form solution of (3) is obtained through representation of the boundary conditions by the Fourier series

$$v(o,t) = a + \Sigma_{n=1}^{\infty}[(2p_n) \cos \omega_n t + (2q_n) \sin \omega_n t] \tag{4a}$$

$$f(o,t) = -K\{b + \Sigma_{n=1}^{\infty}[(4r_n)\zeta_n \cos \omega_n t + (4s_n)\zeta_n \sin \omega_n t]\} \tag{4b}$$

where,

$\omega_n = n\Omega$ and $\zeta_n^2 = \frac{1}{2}\,\omega_n/\kappa$.

We approximated this representation by a least square fit of the boundary data to Fourier sums containing a constant term and the first 20 sine-cosine pairs. Each of the sine-cosine pairs and the constant term can be considered separately. Then the results of these 21 solutions combined by superposition to give the desired solution to (3). In Appendix A we consider the methods for a single sine-cosine pair in detail. For convenience, the subscript, n, is omitted.

Phase 2

In order to obtain the change in temperature resulting from placement near the surface of a disc maintained at a fixed uniform temperature, a set of 7090 programs has been written to solve the heat equation in three space dimensions with cylindrical symmetry using the standard forward difference approximation to the differential equation. Since the temperature difference is to be obtained, the initial condition in the medium is that the temperature difference is zero. The boundary conditions at the surface, however, must reflect the presence of the radiation interchange with the disc and of the solar radiation (during lunar day) and re-radiation of heat into space. These radiation processes depend on the absolute surface temperature which is calculated by superposition of the temperature difference caused by the disc upon the normal, steady periodic solution found in Phase 1. In Appendix B, the numerical process used is described in detail.

Results of Phase 1

Using the 7090 program for Phase 1, a number of sets of temperature vs. time profiles were computed at various depths for various choices of physical parameters. Both uniform and layered media were used. The results indicate a definite limitation on the possible values of the parameters for the medium. If the parameters are physically unrealistic, that is, unrealistic based on our knowledge of and assumptions about the lunar surface, the solutions quickly result in extremely high or extremely low, even negative, temperatures at depths below the surface. Thus, one can establish, within the limits of our assumptions, the maximum depth to which a surface layer of any chosen composition may extend. Below that maximum depth, another type of layer must be assumed to preserve the stability of the solution. It should be noted that in this problem the solution in an upper layer is independent of the choice of lower layers. Of course, the fact that the solutions are well-behaved in an upper layer does not show that it is possible to select lower layers so that the solution will continue to be well behaved.

The following values for parameters were used in all cases:

1. $\alpha_v = 0.875$
2. $q_o = 1.95$ cal/cm^2 min $= 117.0$ cal/cm^2 hr.
3. $\Omega = 1.4776 \times 10^{-4}$ min^{-1} $= 8.8656 \times 10^{-3}$ hr^{-1}.
4. $\epsilon_s = 0.9$
5. $\sigma = 5.669 \times 10^{-12}$ watt/cm^2 $(°K)^4$ $= 4.878 \times 10^{-9}$ cal/cm^2 hr $(°K)^4$.

Lambert's Law was assumed to hold for all surfaces.

Three basic types of material were chosen to make up layers. They were:

1. "Dust*" : $K = 1.65 \times 10^{-5}$ cal/sec cm °K
$= 5.94 \times 10^{-2}$ cal/hr cm °K
$\rho = 2.0$ gram/cm^3
$c = 0.2$ cal/gram °K

*Taken to be dust in vacuum.

2. "Pumice" : $K = 5.37 \times 10^{-4}$ cal/sec cm °K
$= 1.93$ cal/hr cm °K
$\rho = 0.6$ gram/cm^3
$c = 0.2$ cal/gram °K

3. "Basalt" : $K = 5.37 \times 10^{-3}$ cal/sec cm °K
$= 1.93 \times 10^{1}$ cal/hr cm °K
$\rho = 2.8$ gram/cm^3
$c = 0.2$ cal/gram °K

Several sets of two-layered cases were run. In addition, a wide range of conductivities was explored using a uniform medium with ρc the same as "dust". These may show the maximum depths to which such a top layer may be used.

In order to specifically eliminate certain media from the class of "reasonable" media, improved representation of surface data would be required as indicated below.

Results of Phase 2

Most of the calculations done in Phase 2 used the radiation boundary condition corresponding to the disc being placed very close to the surface so that edge effects could be ignored. However, if the disc is not assumed to be very close, allowance must be made for loss of energy by radiation from areas under the disc into the sky. This will cause a more rapid perturbation of the uniformity of the radial surface temperature distribution.

In order to explore this possibility a calculation was made using a radiation condition for a circular disc, parallel to and above the surface. Then, the direct radiation flux from the disc absorbed by the surface is,

$$q_{sh} = \epsilon_s \epsilon_{sh} \sigma T_{sh}^4 \Phi. \tag{5}$$

ϵ_s is the lunar surface emittance, ϵ_{sh} is the disc surface emittance, and T_{sh} is the temperature of the disc. R_{sh} is the radius of the disc, R_s is the radial distance on the surface to the point of interest, h is the height of the disc above the surface, and,

$$\Phi = \frac{1}{2} \{1 - \frac{1 + C^2 - B^2}{\sqrt{C^4 + 2C^2 (1-B^2) + (1+B^2)^2}} \} \tag{6}$$

where $B = R_{sh}/h$ and $C = R_s/h$[3].

For the assumption that the disc is very close (HEIGHT = h = 0.0) to the surface, we use,

$$\Phi = 1, R_s < R_{sh}, \text{ and} \tag{7a}$$

$$\Phi = 0, R_s > R_{sh}. \tag{7b}$$

[3] Heat Transfer, Vol. II, Max Jakob, John Wiley and Sons, New York, 1957, p. 11.

For this case the correct total surface flux for $R_s < R_{sh}$ is given by

$$q_{net} = E\sigma\ (T_{sh}^4 - T_{su}^4) \tag{8}$$

where T_{su} is the surface temperature and

$$E = (1/\epsilon_s + 1/\epsilon_{sh} - 1)^{-1}. \tag{9}$$

Equation (8) includes all reflections and E is the sum of either of the series,

$$E = \epsilon_s \epsilon_{sh} \sum_{m=0}^{\infty} (1-\epsilon_{sh})^m (1-\epsilon_s)^m \tag{10a}$$

$$= \epsilon_s\ [1-(1-\epsilon_{sh})\ \epsilon_s \sum_{m=0}^{\infty} (1-\epsilon_{sh})^m (1-\epsilon_s)^m\ ] \tag{10b}$$

where (10a) represents the reflections applied to T_{sh}^4 and (10b) represents those applied to T_{su}^4.

For $h \neq 0$, the reflection considerations are much more complicated. Using the simple form given by (5) would give results closely comparable to $h = 0$ results if the reflection terms in (10) were neglected. Hence, for $h = 0$ we used only the leading terms of the series (10), thus,

$$q_{net} = \sigma\ (\epsilon_s \epsilon_{sh}\ T_{sh}^4 - \epsilon_s\ T_{su}^4). \tag{11}$$

Several values of the product $K\rho c$ were assigned, and temperature perturbation calculations were made using these values for the case $h = 0$. These results indicate that it would be possible to experimentally determine the product $K\rho c$ for the near-surface lunar material using the apparatus previously described.

The single calculation using $h \neq 0$ indicates that it would be possible to determine $K\rho c$ if the disc were placed at most as far from the surface as 5 cm.

The calculations described above were done for initial time (time of disc placement) = 168.0 hrs. after sunrise. This time is approximately "noon" of the lunar day. In addition, a single calculation was made for "midnight" initial time. All results graphically presented in Appendix D are for "noon" initial time. Although no detailed study of results for "midnight" initial time was made, it is expected that $K\rho c$ could also be determined by an experiment at "midnight". These two initial times were the only ones considered since it seemed desirable to choose an initial time at which the true derivative of normal surface temperatures is very small.

Comments

The application of the methods described above to an actual experiment would, of course, require additional calculation to improve the accuracy of perturbed temperatures given in Appendix D.

No rigorous analysis of errors has been attempted. In particular, no attempt has been made to estimate the effect of using Eqn. (11) rather than Eqn. (8) to express the radiation interchange at the surface.

The surface temperatures used in this work were obtained from a plot entitled, "Temperature Variation of the Lunar Surface", furnished by North American Aviation, Inc. Inaccuracies in $v(o,t)$ are magnified in the surface flux calculated by Eqn. (2), since $f(o,t)$ is small compared to either of the terms $\alpha_v q$ and $\epsilon_s \sigma [v(o,t)]^4$. This inaccuracy leads to a physically unrealistic magnification with depth of the higher frequency components of temperature and flux. In the absence of raw temperature data, no attempt was made to estimate the accuracy of the individual Fourier components for temperature or flux. The 20 sine-cosine pairs were necessary to represent the temperature plot used in the calculations. It is expected that Fourier sums calculated from raw data would justify using fewer components which would significantly improve the results of Phase 1, and possibly affect the results of Phase 2.

In an actual experiment, the initial (normal) surface temperature should be measured to allow correction of the astronomical values previously used. Then perturbed temperatures should be measured at a sequence of times up to approximately one-third of an hour. Then an estimate of the product $K\rho c$ would be determined from an expanded improved version of Fig. 4 for each perturbed temperature measured. An analysis of the variation of these estimates of $K\rho c$ may give some indication of the reliability of the method.

RE-ORDER No. 61-422

In general, the results of this work confirm the preliminary design calculations for surface determination of near-surface thermal properties.

RE-ORDER No. 61-422

APPENDIX A

DETERMINATION OF INITIAL CONDITIONS

To determine initial conditions existing at placement of the disc, consider the problem:

$$v_t = x\, v_{xx}\ ;\ 0 < x < \gamma,\ -\infty < t < \infty, \tag{1a}$$

$$v(0,t) = (2p)\cos \omega t + (2q)\sin \omega t;\ -\infty < t < \infty \tag{1b}$$

$$f(0,t) = -K\frac{\partial v}{\partial x}(0,t) = (-4K\zeta r)\cos \omega t + (-4K\zeta s)\sin \omega t \tag{1c}$$

$$\gamma,\ \omega,\ x,\ K,\ p,\ q,\ r,\ s \text{ constant};\ \zeta^2 = \tfrac{1}{2}\,\omega/x,\ \omega \neq 0. \tag{1d}$$

$$\begin{aligned} v(x,t) = {} & [P\cos(\omega t+\zeta x) + Q\sin(\omega t+\zeta x)]\, e^{\zeta x} \\ & + [R\cos(\omega t-\zeta x) + S\sin(\omega t-\zeta x)]\, e^{-\zeta x}, \end{aligned} \tag{2a}$$

satisfies (1a) (Appendix C.I). Hence, if constants P, Q, R, S can be chosen satisfying (1b) and (1c), then v(x,t) given by (2a) will satisfy (1).

Differentiating (2a) we obtain,

$$\begin{aligned} f(x,t) = -K\frac{\partial v}{\partial x} = -K\zeta\{ & [-P\sin(\omega t+\zeta x) + P\cos(\omega t+\zeta x) \\ & + Q\cos(\omega t+\zeta x) + Q\sin(\omega t+\zeta x)]\, e^{\zeta x} \\ & - [-R\sin(\omega t-\zeta x) + R\cos(\omega t-\zeta x) \\ & + S\cos(\omega t-\zeta x) + S\sin(\omega t-\zeta x)]\, e^{-\zeta x}\}. \end{aligned} \tag{2b}$$

Requiring (2a) and (2b) to reduce to (1b) and (1c), respectively for $x = 0$, we obtain,

$$2p = P + R;\ 2q = Q + S, \tag{3a}$$

RE-ORDER No. 61-422

and $4r = P + Q - R - S$; $4s = -P + Q + R - S$, (3b)

or $2(r+s) = Q - S$; $2(r-s) = P - R$. (3c)

$$\therefore, \quad \begin{aligned} P &= p + r - s \\ Q &= q + r + s \\ R &= p - r + s \\ S &= q - r - s \end{aligned} \qquad (4)$$

We wish to write (2) in the form

$$v(x,t) = P_I^* (x) \cos \omega t + Q_I^* (x) \sin \omega t, \qquad (5a)$$

$$f(x,t) = -K\zeta \{R_I^* (x) \cos \omega t + S_I^* (x) \sin \omega t\}. \qquad (5b)$$

To do so we apply trigonometric addition formulae to (2) obtaining (Appendix C.II, C.III)

$$\begin{aligned} P_I^* (x) &= P^* (x) + R^* (x) \\ Q_I^* (x) &= Q^* (x) + S^* (x) \\ R_I^* (x) &= Q^* (x) - R^* (x) + P^* (x) - S^* (x) \\ S_I^* (x) &= Q^* (x) + R^* (x) - P^* (x) - S^* (x) , \end{aligned} \qquad (6)$$

where

$$\begin{aligned} P^* (x) &= (P \cos \zeta x + Q \sin \zeta x) e^{\zeta x} \\ Q^* (x) &= (-P \sin \zeta x + Q \cos \zeta x) e^{\zeta x} \\ R^* (x) &= (R \cos \zeta x - S \sin \zeta x) e^{-\zeta x} \\ S^* (x) &= (R \sin \zeta x + S \cos \zeta x) e^{-\zeta x} \end{aligned} \qquad (7)$$

Given data $\quad v(0,t) = P_D \cos \omega t + Q_D \sin \omega t$ (8a)

$\quad f(0,t) = R_D \cos \omega t + S_D \sin \omega t$ (8b)

1:794.38-A2

We compute
$$p = \frac{1}{2} P_D \; , \; q = \frac{1}{2} Q_D \quad : \qquad (9)$$
$$r = -\frac{1}{4K\zeta} R_D , \; s = -\frac{1}{4K\zeta} S_D$$

Then, P, Q, R, S are given by (4). Then, (2) gives temperature and flux throughout $0 < x < \gamma$, $-\infty < t < \infty$. Alternatively, for fixed $x \in (0,\gamma)$, (5) gives temperature and flux for all time.

In particular, if $x = \gamma$ represents an interface between two layers;

$$P_I^*(\gamma),\; Q_I^*(\gamma),\; -K_1\zeta_1 R_I^*(\gamma),\; -K_1\zeta_1 S_I^*(\gamma), \qquad (10)$$

respectively play the role of

$$P_D,\; Q_D,\; R_D,\; S_D$$

for computation in a second layer by reduction to the previous case where K_1 is the conductivity appropriate to the first layer. Distance, x, in the second layer will be measured from the interface rather than from the top of the first layer. Clearly, this process may be continued for as many layers as desired.

The process has two parts, viz.

1° We <u>cross</u> an interface, i.e., we apply (4) to data representing input at the top of a layer.

2° We use coefficients so obtained to <u>carry</u> ourselves into or through a layer by application (6) and (7) to obtain (5).

In the computer, (7a) for example, may be difficult to evaluate for the desired x due to overflow on $e^{\zeta x}$; even when $P^*(x)$ itself is not over-large. Hence, it may be desirable to pretend the existence of certain interfaces, i.e., we may represent a single thick layer by several thinner layers of identical physical properties to guarantee smallness of ζx throughout every layer (recall x is always measured from the top of the layer of interest).

In the special case, $\omega = 0$, the above formulation does not apply. That is, given data

$$v(0,t) = a; \quad f(0,t) = -Kb \qquad \begin{matrix}(1b1)\\(1c1)\end{matrix}$$

we find the solution

$$v(x,t) = a + bx \qquad (2a1)$$

$$f(x,t) = -Kb \qquad (2b1)$$

Now, if we are given data

$$v(0,t) = \Sigma_{n=o}^{N} \{P_{D_n} \cos \omega_n t + Q_{D_n} \sin \omega_n t\} \qquad (8a1)$$

$$f(0,t) = \Sigma_{n=o}^{N} \{R_{D_n} \cos \omega_n t + S_{D_n} \sin \omega_n t\}, \qquad (8a2)$$

$$\omega_n = n\Omega$$

We simply apply the above results by superposition to obtain our solution. Note that $a = P_{D_o}$, $-Kb = R_{D_o}$.

APPENDIX B

DETERMINATION OF TEMPERATURE PERTURBATIONS

In Appendix A we dealt with the determination of normal lunar temperatures, i.e., those temperatures which exist in the absence of disturbance by man-made equipment.

These normal temperatures satisfy the heat equation in one space dimension. Since this temperature is known for all time and independent of radial distance, r, measured from the center of the disc and azimuth, θ, the following equations are satisfied:

$$\frac{\partial v}{\partial t} = \kappa\{\frac{\partial^2 v}{\partial x^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2}\} \; ; \; r, x > 0, -\infty < t < \infty \qquad (1a)$$

$$\frac{\partial v}{\partial r}(x,r,t) \equiv 0, \text{ in particular } \frac{\partial v}{\partial r}(x,o,t) = 0; \; t, x > 0 \qquad (1b)$$

$$\frac{\partial v}{\partial x}(o,r,t) \text{ - known for all t.} \qquad (1c)$$

$$v(x,r,o) = v(x,r,o) \text{ - known by Phase 1} \qquad (1d)$$

$$v(o,r,t) \text{ - known from all t} \qquad (1e)$$

where time, $t = 0$, is naturally taken as the time of placement of the disc.

Now after placement of the disc, i.e., for positive t, the above relations will not describe the temperature. Rather, the temperature $\mu(x,r,t)$ will satisfy:

$$\frac{\partial \mu}{\partial t} = \kappa\{\frac{\partial^2 \mu}{\partial x^2} + \frac{1}{r}\frac{\partial \mu}{\partial r} + \frac{\partial^2 \mu}{\partial r^2}\}; \; x,r,t > 0, \qquad (1a1)$$

RE-ORDER No. 61-422

$$\frac{\partial \mu}{\partial r}(x,o,t) = 0; \; t, \; x > 0 \text{ by cylindrical symmetry,} \tag{1b1}$$

$$-K\frac{\partial \mu}{\partial x}(o,r,t) = F(r,t) - \eta[\mu(o,r,t)]^4 \; ; \; r,t > 0, \tag{1c1}$$

$$\mu(x,r,o) = v(x,r,o). \tag{1d1}$$

Since $v(x,r,t)$ is known in closed form from Phase 1, we consider for simplicity in calculation:

$$w(x,r,t) = \mu(x,r,t) - v(x,r,t). \tag{2}$$

From the above definition of μ and v, we have

$$\frac{\partial w}{\partial t} = \kappa\{\frac{\partial^2 w}{\partial x^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2}\} \; ; \; x,r,t > 0 \tag{1a2}$$

$$\frac{\partial w}{\partial r}(x,o,t) = 0 \; ; \; t,x > 0 \tag{1b2}$$

$$-K\frac{\partial w}{\partial x}(o,r,t) = F(r,t) - \eta\mu^4(o,r,t) + K\frac{\partial v}{\partial x}(o,r,t); \; r,t > 0, \tag{1c2}$$

$$w(x,r,o) \equiv 0. \tag{1d2}$$

Then, $w(x,r,t)$ represents perturbation of temperature from normal due to the placing of the disc. $w(x,r,t)$ was computed numerically as described below:

Let $Z_{ijn} = Z(i\Delta x, j\Delta r, n\Delta t)$ represent the approximation to $w(i\Delta x, j\Delta r, n\Delta t)$ obtained below. Let the operator L be defined by

$$LZ_{ijn} = (\Delta x)^2 \; [\Delta_x^2 Z_{ijn} + \frac{Z_{i,j+1,n} - Z_{i,j-1,n}}{2j(\Delta r)^2} + \Delta_r^2 Z_{ijn}] \tag{3}$$

where

$$\Delta_x^2 Z_{ijn} = (Z_{i+1,j,n} - 2Z_{i,j,n} + Z_{i-1,j,n})/(\Delta x)^2. \tag{4}$$

It seems desirable to take $\Delta x = \Delta r$ and we assume this to be done.

We define Z_{ijn} by the difference equations (Appendix C.IV)

$$Z_{i,j,n+1} = Z_{ijn} + \frac{\kappa \Delta t}{(\Delta x)^2} LZ_{ijn} \; ; \; n \geq 0; \; x,r > 0, \tag{1a3}$$

$$Z_{i,o,n} = Z_{i,1,n} \; ; \; n \geq 0, \; i > 0, \tag{1b3}$$

$$Z_{o,j,n} = \phi_{o,j,n} - v_{o,j,n} \; ; \; n \geq 0, \; j > 0 \tag{1c3}$$

where $\phi_{o,j,n}$ is the positive root of

$$\left(\frac{2}{3} \frac{\eta}{K} \Delta x\right) \phi^4 + \phi - \{v_{o,j,n} + \frac{1}{3} [4Z_{1,j,n} - Z_{2jn} + 2\Delta x \left(\frac{F_{j,n}}{K} + \frac{\partial v}{\partial x} o,j,n\right)]\} = 0 \tag{5}$$

$$Z_{i,j,o} = 0. \tag{1d3}$$

Given values at $t = n\Delta t$, we compute

$Z_{i,j,n+1}$ by (1a3); $ij = 1,2, \ldots$.
$Z_{i,o,n+1}$ by (1b3); $i = 1,2, \ldots$.
$Z_{o,j,n+1}$ by (1c3); $j = 1,2, \ldots$.

$Z_{o,o,n+1}$ could be computed by either (1b3) or (1c3), but is immaterial since it is not used for progress to the next time level. However, it might be of interest to compare these two values of $Z_{o,o,n}$ for large n.

RE-ORDER No. 61-422

Algebraic Problem: Denote $A = \frac{2\eta}{3K} \Delta x$ and $B_{jn} = v_{ojn} + \frac{1}{3} [4Z_{1,j,n}$

$$- Z_{2jn} + 2\Delta x \frac{F_{jn}}{K} + \frac{\partial v}{\partial x} {}_{ojn})]. \qquad (6)$$

Now, (5) has precisely <u>one</u> positive root if $B_{jn} > 0$ and <u>no</u> positive root if $B_{jn} \leq 0$, since the function $g(\phi) = A\phi^4 + \phi - B$ is negative at $\phi = 0$ for positive B and positive for negative B; $g'(\phi) = 4A\phi^3 + 1$ is positive for all positive ϕ; and $g(\phi)$ is positive for sufficiently large ϕ.

Thus, we have Z_{ijn} completely defined. For stable choices of Δx, Δr, Δt, B is always positive. In fact, $v_{o,j,n}$ is normally the dominant term in B for sufficiently small Δx.

Although μ_{ijn}, especially μ_{ojn}, is desirable output, Z_{ijn} has obvious computer storage advantages. In fact, Z_{ijn} at time $t = n\Delta t$ is defined to be zero for all sufficiently large i and j. Thus, Z_{ijn} requires much less storage than μ_{ijn}. Even so, the magnitude of the problem which may be solved without use of magnetic tape for interim storage is rather small. We made no provision for this interim storage; hence, our results will be given for times of, at most, one hour.

Throughout the computations we require $\Delta x = \Delta r$ and Δt to be chosen to satisfy

$$\frac{x\Delta t}{(\Delta x)^2} < \frac{1}{4} \qquad (7)$$

to maintain numerical stability[1].

[1] Finite Difference Methods for Partial Differential Equations, G. E. Forsythe, W. R. Wasow, Wiley, 1960

Most references give 1/2 where we have given 1/4. However, the proper expression depends on the number of space dimensions actually appearing in the heat equation. 1/2 is correct for one-space dimension, while 1/4 is correct for two dimensions (x and r).

The above considerations are altered by the presence of an interface between layers. If $i\Delta x$ is a depth corresponding to an interface we determine Z_{ijn+1} by the expression

$$(K_1+K_2)\, Z_{i,j,n+1} = K_2\, Z_{i+1,j,n+1} + K_1\, Z_{i-1,j,n+1}\,, \qquad (8)$$

which approximates

$$K_1 \frac{\partial w}{\partial x}(x-o,r,t) = K_2 \frac{\partial w}{\partial x}(x+o,r,t) \qquad (9a)$$

$$w(x-o,r,t) = w(x+o,r,t), \qquad (9b)$$

where K_1 and K_2 are conductivities of the first and second layers, respectively. Otherwise, relations (1a3), (1b3), (1c3), and (1d3) apply. In this case the diffusivity, $\varkappa$, appearing in the stability requirement (7) is the largest diffusivity encountered in the problem.

APPENDIX C

MATHEMATICAL NOTES

I. v(x,t) given by (A2a) satisfies (A1a):

$$\kappa\frac{\partial^2}{\partial x^2}\{e^{\pm\zeta x}\cos(\omega t\pm\zeta x)\} = \kappa(\pm\zeta)\frac{\partial}{\partial x}\{e^{\pm\zeta x}[-\sin(\omega t\pm\zeta x)$$

$$+\cos(\omega t\pm\zeta x)]\} = \kappa\zeta^2 e^{\pm\zeta x}\{-\sin(\omega t\pm\zeta x)+\cos(\omega t\pm\zeta x)$$

$$-\cos(\omega t\pm\zeta x)-\sin(\omega t\pm\zeta x)\} = \frac{1}{2}\omega e^{\pm\zeta x}\{-2\sin(\omega t\pm\zeta x)\}$$

$$= -\omega e^{\pm\zeta x}\sin(\omega t\pm\zeta x) = \frac{\partial}{\partial t}\{e^{\pm\zeta x}\cos(\omega t\pm\zeta x)\}.$$

Since $\sin(\omega t\pm\zeta x) = \cos(\omega t\pm\zeta x-\pi/2)$, $e^{\pm\zeta x}\sin(\omega t\pm\zeta x)$ also satisfies (A1a); hence, v(x,t) satisfies (A1a).

II. Addition formulae applied to v(x,t):

By (A2a),
$$v(x,t) = \{[P(\cos\omega t\cos\zeta x-\sin\omega t\sin\zeta x)+Q(\sin\omega t\cos\zeta x+\cos\omega t\sin\zeta x)]e^{\zeta x}+[R(\cos\omega t\cos\zeta x+\sin\omega t\sin\zeta x)+S(\sin\omega t\cos\zeta x-\cos\omega t\sin\zeta x)]e^{-\zeta x}\}$$

$$= \{[(P\cos\zeta x+Q\sin\zeta x)e^{\zeta x}+(R\cos\zeta x-S\sin\zeta x)e^{-\zeta x}]\cos\omega t+[(-P\sin\zeta x+Q\cos\zeta x)e^{\zeta x}+(R\sin\zeta x+S\cos\zeta x)e^{-\zeta x}]\sin\omega t\}$$

$$= [P^*(x)+R^*(x)]\cos\omega t+[Q^*(x)+S^*(x)]\sin\omega t.$$

III. Addition formulae applied to $f(x,t)$:

We obtain the desired result by differentiating $v(x,t)$ since application of addition formulae is tedious. From (A5a) and (A7),

$$f(x,t) = -K\frac{\partial v}{\partial x} = -K\zeta\{[Q^*(x) - R^*(x) + P^*(x) - S^*(x)]\cos\omega t$$
$$+ [Q^*(x) + R^*(x) - P^*(x) - S^*(x)]\sin\omega t\}$$

since

$$\frac{dP^*}{dx} = \zeta[P^*(x) + Q^*(x)],$$

$$\frac{dQ^*}{dx} = \zeta[Q^*(x) - P^*(x)],$$

$$\frac{dR^*}{dx} = \zeta[-R^*(x) - S^*(x)], \text{ and}$$

$$\frac{dS^*}{dx} = \zeta[-S^*(x) + R^*(x)].$$

IV. Divided Differences: Let $w = w(x,r,t)$ and $w_{ijn} = w(i\Delta x, j\Delta r, n\Delta t)$; then

$$w_{i,j+1,n} = w_{ijn} + \Delta r \frac{\partial w}{\partial r}_{ijn} + \frac{(\Delta r)^2}{2}\frac{\partial^2 w}{\partial r^2}_{ijn} + \frac{(\Delta r)^3}{6}\frac{\partial^3 w}{\partial r^3}_{ijn} + 0((\Delta r)^4),$$

$$w_{i,j-1,n} = w_{ijn} - \Delta r \frac{\partial w}{\partial r}_{ijn} + \frac{(\Delta r)^2}{2}\frac{\partial^2 w}{\partial r^2}_{ijn} - \frac{(\Delta r)^3}{6}\frac{\partial^3 w}{\partial r^3}_{ijn} + 0((\Delta r)^4).$$

$$\therefore,\quad w_{i,j+1,n} - w_{i,j-1,n} = 2\Delta r \frac{\partial w}{\partial r}_{ijn} + 0((\Delta r)^3).$$

Also, $w_{i,j+1,n} + w_{i,j-1,n} = 2w_{ijn} + (\Delta r)^2 \frac{\partial^2 w}{\partial r^2}_{ijn} + 0((\Delta r)^4).$

Similarly, $w_{i+1,j,n} + w_{i-1,j,n} = 2w_{ijn} + (\Delta x)^2 \frac{\partial^2 w}{\partial x^2} ijn + O((\Delta x)^4).$

$$w_{2jn} = w_{ojn} + 2\Delta x \frac{\partial w}{\partial x} ojn + \frac{4(\Delta x)^2}{2} \frac{\partial^2 w}{\partial x^2} ojn + O((\Delta x)^3),$$

$$w_{1jn} = w_{ojn} + \Delta x \frac{\partial w}{\partial x} ojn + \frac{(\Delta x)^2}{2} \frac{\partial^2 w}{\partial x^2} ojn + O((\Delta x)^3).$$

$\therefore$, $w_{2jn} - 4w_{1jn} = -3\, w_{ojn} - 2\Delta x \frac{\partial w}{\partial x} ojn + O((\Delta x)^3).$

$$Lw_{ijn} = (\Delta x)^2 \{\Delta_x^2 w_{ijn} + \frac{w_{i,j+1,n} - w_{i,j-1,n}}{2r\ \Delta r} + \Delta_r^2 w_{ijn}\}$$

$$= (\Delta x)^2 \{\frac{\partial^2 w}{\partial x^2} ijn + O((\Delta x)^2) + \frac{1}{r} [\frac{\partial w}{\partial r} ijn + O((\Delta r)^2)]$$

$$+ \frac{\partial^2 w}{\partial r^2} ijn + O((\Delta r)^2)\}$$

$$= (\Delta x)^2 \{w_{xx} + r^{-1} w_r + w_{rr}\} + O((\Delta x)^4 + \frac{(\Delta r)^2 (\Delta x)^2}{r}$$

$$+ (\Delta x)^2 (\Delta r)^2)$$

$$= \frac{(\Delta x)^2}{x} w_t + O((\Delta x)^4 + \frac{(\Delta r)^2 (\Delta x)^2}{r} + (\Delta x)^2 (\Delta r)^2)$$

$$= \frac{(\Delta x)^2}{x} \{\frac{w_{i,j,n+1} - w_{i,j,n}}{\Delta t} + O(\Delta t)\} + O((\Delta x)^4 + \frac{(\Delta r)^2 (\Delta x)^2}{r}$$

$$+ (\Delta x)^2 (\Delta r)^2).$$

$\therefore$, $\frac{x\Delta t}{(\Delta x)^2} Lw_{ijn} = w_{i,j,n+1} - w_{ijn} + O((\Delta t)^2 + \Delta t(\Delta x)^2 + \frac{\Delta t(\Delta r)^2}{r}$

$$+ \Delta t(\Delta r)^2).$$

APPENDIX D

SUMMARY OF NUMERICAL RESULTS

All calculations were based on the temperature and flux Fourier coefficients given below. The flux coefficients give flux units cal/sec cm^2, hence, the 7090 programming multiples flux coefficients by 3600.0 to obtain flux units cal/hr cm^2. Since flux depends on the parameters shown above (Eqn. 4), it would be necessary to recompute flux coefficients if the parameters were to be altered.

Time profiles of normal temperatures in "dust" are given below for several depths (Fig. 1). For other media, the general form of the time profiles is very similar. This is indicated by application of the transformation

$$v(x,t) \equiv V(\xi,t) \text{ where } \xi = \int_0^x \frac{d\alpha}{K(\alpha)} \tag{1}$$

to Equations A1, obtaining

$$\frac{\partial V}{\partial t} = \frac{x}{K^2} \frac{\partial^2 V}{\partial \xi^2} = \frac{1}{K\rho c} \frac{\partial^2 V}{\partial \xi^2} \tag{2a}$$

$$V(o,t) = v(o,t) \tag{2b}$$

$$f(o,t) = - \frac{\partial V}{\partial \xi}(o,t). \tag{2c}$$

Thus, the time profiles in the (ξ,t) coordinate system depend only on the product $K\rho c$.

Curves representing perturbation of temperatures for several values of $K\rho c$ are presented in Fig. 2. Since these curves were obtained using height of disc = HEIGHT = 0.0, no angular effect (radiation into space) is considered in these curves. For comparison, one case was computed for HEIGHT = 5.0 cm. The results are compared in Fig. 3. Note that the results indicate that the radial variation of perturbed temperature for a specific time is small over a radius of 4 or 5 cm. Thus, the perturbed temperature is nearly independent of radius over the area of view of the radiometer.

For sufficiently small time so that the radius of the disc has little effect, $(K\rho c)^{-1/2}$ can be considered a function of the temperature perturbations. Thus, if the surface temperature perturbation, $w(o,o,t)$, is known for a definite value of t, $(K\rho c)^{-1/2}$ can be estimated as indicated in Fig. 4.

Since the Phase 2 calculations used finite-difference techniques, there is some error in the temperature perturbations obtained. Although no rigorous analysis of errors has been made, an indication of errors is obtained by comparison of results for successively smaller Δt and Δr. The temperature perturbations obtained in "dust" using $\Delta x = \Delta r = 0.1$ cm, 0.05 cm, 0.025 cm, successively, differ by 1°K or less where Δt is chosen to maintain numerical stability.

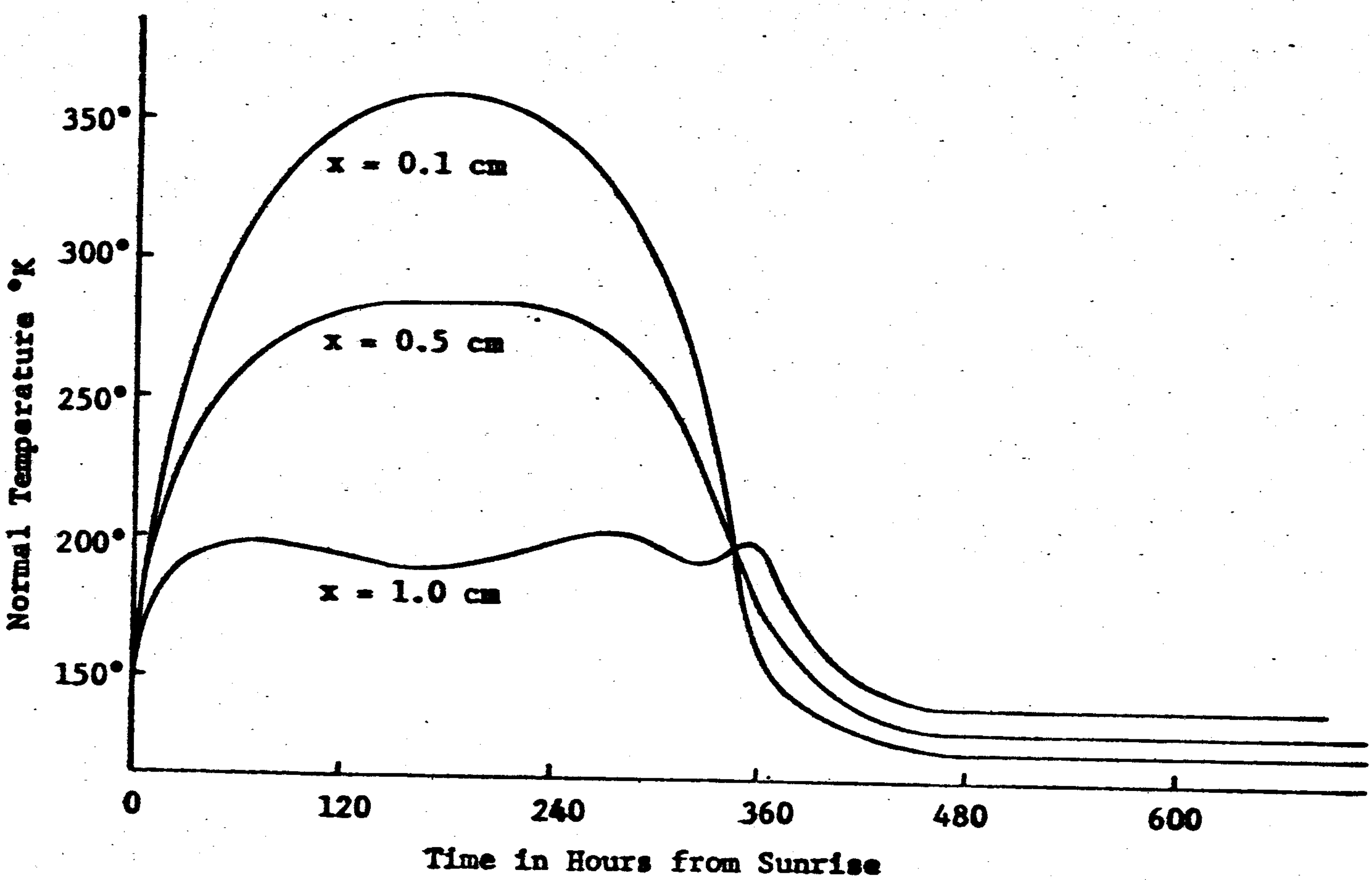
TABLE I

FOURIER COEFFICIENTS

N	Temp. Sine	Temp. Cos	Flux Sine	Flux Cos	
	DATA				
00	.00000000E 00	.22626791E 03	.00000000E 00	.91183791E-03	LUNAR1
01	.14474085E 03	-.52482391E 01	.18406930E-02	.11056784E-03	LUNAR1
02	.16495898E 01	-.34031582E 02	.20146566E-04	-.63172640E-03	LUNAR1
03	.22540356E 02	-.33538482E 01	.21957966E-03	.44425867E-04	LUNAR1
04	.20174133E 01	-.13186581E 02	-.13201978E-04	-.21278920E-03	LUNAR1
05	.84914122E 01	-.19856164E 01	.11512204E-03	.36994564E-04	LUNAR1
06	.15525196E 01	-.76848104E 01	-.90358427E-05	-.13197542E-03	LUNAR1
07	.43425736E 01	-.12548302E 01	.59816251E-04	.28458076E-04	LUNAR1
08	.11317164E 01	-.52154533E 01	-.26333029E-05	-.85040427E-04	LUNAR1
09	.26702678E 01	-.90534500E 00	.37523709E-04	.21839815E-04	LUNAR1
10	.78925178E 00	-.36548020E 01	.31907033E-05	-.62879271E-04	LUNAR1
11	.19471356E 01	-.79391820E 00	.23770127E-04	.16761790E-04	LUNAR1
12	.62367758E 00	-.31022627E 01	.67389333E-05	-.45032700E-04	LUNAR1
13	.13407852E 01	-.79024720E 00	.19345902E-04	.11874344E-04	LUNAR1
14	.37446378E 00	-.26508531E 01	.94848804E-05	-.37742956E-04	LUNAR1
15	.90879622E 00	-.69753089E 00	.13012129E-04	.51275089E-05	LUNAR1
16	.22515756E 00	-.21995335E 01	.62070178E-05	-.31778413E-04	LUNAR1
17	.50279911E 00	-.62417482E 00	.90380011E-05	.48891571E-05	LUNAR1
18	.51452000E-01	-.17863406E 01	.74823758E-05	-.25173496E-04	LUNAR1
19	.34896822E 00	-.47163624E 00	.90437187E-05	.17235374E-05	LUNAR1
20	.23009778E-01	-.14265451E 01	.60155256E-05	-.24689393E-04	LUNAR1
-01	.00000000E 00	.00000000E 00	.00000000E 09	.00000000E 00	LUNAR1

1:794.38-D3

RE-ORDER No. 61-422

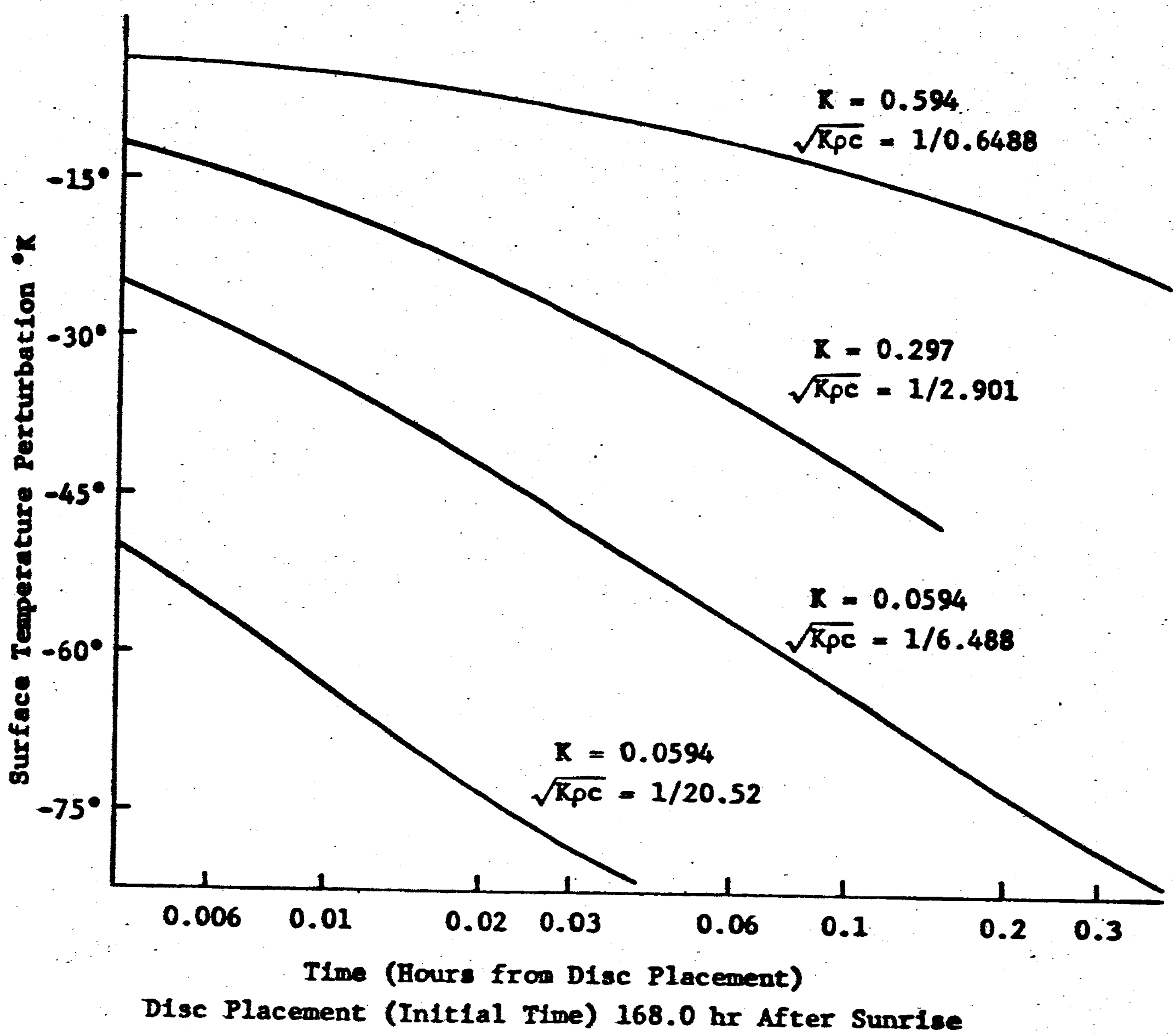


TIME PROFILES OF NORMAL TEMPERATURES IN "DUST"

FIGURE 1

1:794.38-D4
1:794-86

RE-ORDER No. 61-422

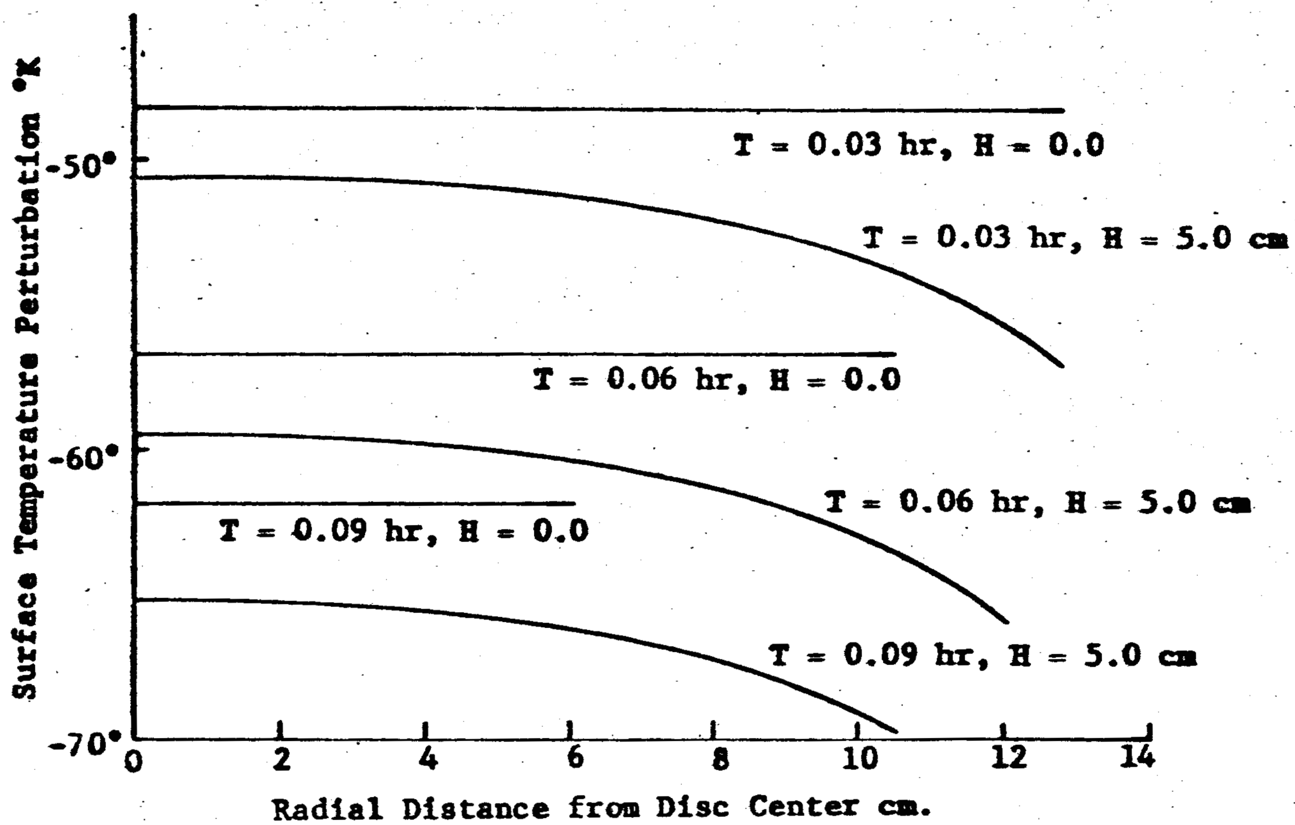


SURFACE TEMPERATURE PERTURBATION FOR VARIOUS UNIFORM MEDIA

FIGURE 2

1:794.38-D5
1:794-87

RE-ORDER No. 61-422



Disc Placement (Initial Time) 168.0 hr. after Sunrise
Results for "Dust" using $\Delta x = \Delta r = 0.1$ cm.

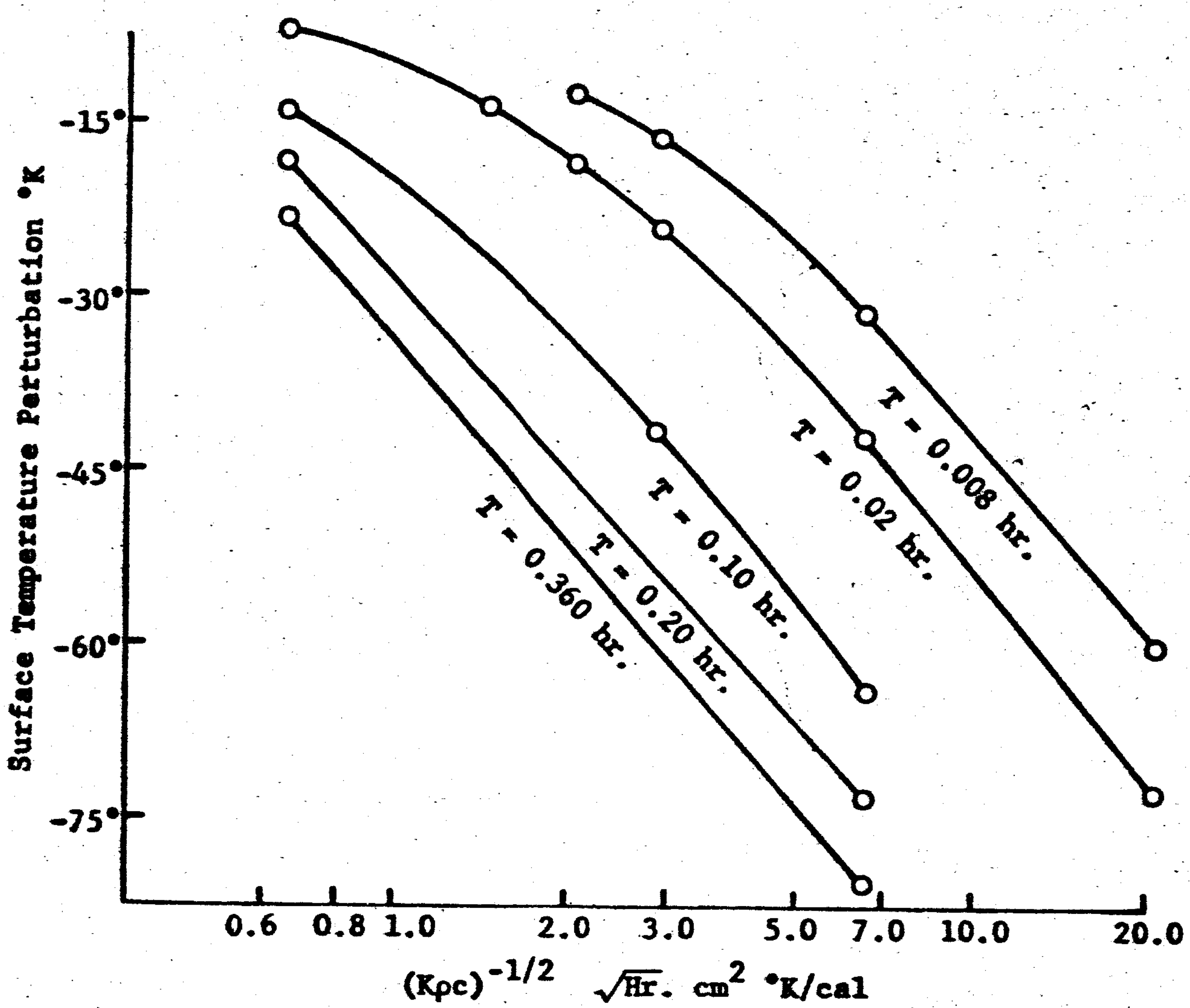
Radius of Disc 15.24 cm.

EFFECT OF HEIGHT (H) VARIATION ON PERTURBATIONS VS. RADIAL DISTANCE

FIGURE 3

1:794.38-D6
1:794-88

RE-ORDER No. 61-42



Disc Placement (Initial Time) 168.0 hr.
After Sunrise

Results using $\Delta x = \Delta r = 0.05$ cm. Radius of Disc = 15.24 cm.
Height of Disc = 0.0

DETERMINATION OF $\sqrt{K\rho c}$ FROM PERTURBED TEMPERATURES

FIGURE 4

1:794.38-D7
1:794-89

APPENDIX E

7090 PROGRAMS

All programming and computation for the preparation of this report was done on Texaco IBM 7090, Houston, Texas with the exception of the determination of Fourier coefficients, which was accomplished on the Texaco Elecom 120A, Bellaire, Texas.

Fortran source listings of this 7090 programming appear below.

These programs operated effectively in the solution of those problems undertaken in the preparation of this report. However, no claim to programming perfection is made. In fact, certain improvements are known to be possible.

The Fortran statement, COMMON, does not appear in Phase 1. This statement is used extensively in Phase 2. In fact, most portions of Phase 2 contain a complete set of Phase 2 COMMON and DIMENSION statements regardless of necessity. These statements are designed to be useful in Phase 1 if Phase 1 were appropriately revised.

Storage requirements listed below are given as decimal numbers. A glossary of variable names is given on Page 40 of this Appendix.

RE-ORDER No. 61-42

Entry Name	- LUNAR1
Category	- Main Program - Phase 1
Purpose	- To read data and guide output of normal temperatures as specified by input control.
Arguments	- None (Main Program)
Unusual Cautions	- None
Description	- After input has been read, calculation is made as described in Appendix A using appropriate subroutines as needed.
Lower Memory Requirements	- 1613
Transfer Vector	- (FPT), (STH), (FIL), (TSH), (RTN), FILET, FILEX, EXIT
Common Requirements	- None

RE-ORDER No. 61-422

```
*       LIST
*       LABEL
CLUNAR1   ALPHABET FOR LABEL
 90     FORMAT (1X,1I3,4E14.8)
 92     FORMAT (1X,1I3,2E14.8,1I4)
 93     FORMAT (1H1,27HLUNAR1 TEMPERATURE PROFILES)
 94     FORMAT (1H0,18HPHYSICAL CONSTANTS)
 95     FORMAT (1H ,1I3,4E14.8)
 96     FORMAT (1H0,17HTIME PROFILE. X= ,E14.8,12H  DELTA T = ,E14.8)
 97     FORMAT (1H0,18HDEPTH PROFILE. T= ,E14.8,12H  DELTA X = ,E14.8)
        DIMENSION N(25),CP(25),CQ(25),CR(25),CS(25),DUSE(5),L(5)
        DIMENSION ANS(999),XP(25),XQ(25),XR(25),XS(25),CON(5),THIC(5)
        DIMENSION DP(25),DQ(25),DR(25),DS(25),RHO(5),SPEH(5)
        K=0
        WRITE OUTPUT TAPE 3,93
 1      K=K+1
        READ INPUT TAPE 2, 90, N(K), DQ(K), DP(K), DS(K), DR(K)
        DS(K)=3600.0*DS(K)
        DR(K)=3600.0*DR(K)
        IF (N(K)) 2,1,1
 2      K=0
        WRITE OUTPUT TAPE 3,94
 3      K=K+1
        READ INPUT TAPE 2,90,L(K),CON(K),THIC(K),RHO(K),SPEH(K)
        WRITE OUTPUT TAPE 3,95,L(K),CON(K),THIC(K),RHO(K),SPEH(K)
        IF (L(K)) 8,9,11
 9      DUSE(K)=CON(K)/(RHO(K)*SPEH(K))
        GO TO 3
 11     DUSE(K)=CON(K)/(RHO(K)*SPEH(K))
 4      READ INPUT TAPE 2,92,KFIL,FIX,SPACE,NREP
        DO 10 I=1, 25, 1
        CP(I) = DP(I)
        CQ(I) = DQ(I)
        CR(I) = DR(I)
        CS(I) = DS(I)
 10     CONTINUE
        IF (KFIL) 5,6,7
 5      GO TO 2
 6      WRITE OUTPUT TAPE 3,96,FIX,SPACE
        CALL FILET (CP,CQ,CR,CS,XP,XQ,XR,XS,DUSE,CON,THIC,
       1            ANS,N,L,FIX,SPACE,NREP)
        GO TO 4
 7      WRITE OUTPUT TAPE 3,97,FIX,SPACE
        CALL FILEX (CP,CQ,CR,CS,XP,XQ,XR,XS,DUSE,CON,THIC,
       1            ANS,N,L,FIX,SPACE,NREP)
        GO TO 4
 8      CALL EXIT
        END
```

1:794.38-E3

Entry Name	- FILET
Category	- Subroutine - Phase 1 only.
Purpose	- To calculate and report as output a profile of normal temperatures vs. time for fixed depth.
Arguments	- CP, CQ, CR, CS, XP, XQ, XR, XS, DUSE, CON, THIC, ANS, N, L, FIX, SPACE, NREP L : The vector L(M) which indicates type of definition for thickness of each layer M, FIX : The fixed depth for the profile, SPACE: Time between successive values in the profile, NREP : Number of values in the profile beginning with value at T = 0. All other arguments appear in the glossary.
Error Exits	- #5 - 7090 Failure
Description	- Using the subroutines CROSS and CARRY, FILET computes a time profile as described in Appendix A.
Lower Memory Requirements	- 372
Transfer Vector	- ERRORQ, CROSS, CARRY, SIN, COS, (STH), (FIL).
Common Requirements	- None

```
*       LIST
*       LABEL
        SUBROUTINE FILET (CP,CQ,CR,CS,XP,XQ,XR,XS,DUSE,CON,
       1                 THIC,ANS,N,L,FIX,SPACE,NREP)
 90     FORMAT (1H , 10E12.5)
        DIMENSION N(25),CP(25),CQ(25),CR(25),CS(25),DUSE(5),L(5)
        DIMENSION ANS(999),XP(25),XQ(25),XR(25),XS(25),CON(5),THIC(5)
        NREP=NREP
        DO 1 I=1,NREP,1
        ANS(I)=0.0
 1      CONTINUE
        K=0
 2      M=0
        K=K+1
        IF (N(K)) 9,8,8
 8      X=FIX
        TERM=N(K)
 3      M=M+1
        XCAR=X
        IF (L(M)) 4,5,6
 4      CALL ERRORQ (5,2)
 5      IF (X-THIC(M)) 6,7,7
 7      XCAR=THIC(M)
 6      CALL CROSS (XP(K),XQ(K),XR(K),XS(K),CP(K),CQ(K),CR(K),CS(K),
       1            DUSE(M),CON(M),EGA,ZETA,TERM)
        CALL CARRY (XP(K),XQ(K),XR(K),XS(K),CP(K),CQ(K),CR(K),CS(K),
       1            CON(M),EGA,ZETA,XCAR)
        IF (L(M)) 4,13,11
 13     IF (X-THIC(M)) 11,11,10
 10     X=X-THIC(M)
        GO TO 3
 11     TAU=0.0
        DO 12 I=1,NREP,1
        ANS(I)=ANS(I)+CP(K)*COSF(EGA*TAU)+CQ(K)*SINF(EGA*TAU)
        TAU=TAU+SPACE
 12     CONTINUE
        GO TO 2
 9      DO 14 I=1,NREP,10
        K=I+9
        WRITE OUTPUT TAPE 3,90,(ANS(J),J=I,K)
 14     CONTINUE
        RETURN
        END
```

Entry Name	- FILET (modified for plot of output)
Purpose	- Same as FILET (E4).
Category	- Subroutine - Phase 1 only.
Arguments	- CP, CQ, CR, CS, XP, XQ, XR, XS, DUSE, CON, THIC, ANS, N, L. FIX, SPACE, NREP L : The vector L(M) which indicates type of definition for thickness of each layer M, FIX : The fixed depth for the profile, SPACE: Time between successive values in the profile, NREP : Number of values in the profile beginning with value at T = 0. All other arguments appear in the glossary.
Error Exits	- #5 - 7090 failure.
Description	- Using the subroutines CROSS and CARRY, FILET computes a time profile as described in Appendix A.
Name of Deck	- FILTPL - the output is plotted by UMPLOT, University of Michigan Plot, available through SHARE.
Lower Memory Requirements	- 2300 (approximately)
Transfer Vector	- ERRORQ, CROSS, CARRY, SIN, COS, (STH), (FIL), PLOT1, PLOT2, PLOT3, FPLOT4
Common Requirements	- None

```
*      LIST
*      LABEL
CFILTPL      FILET WITH PLOT
       SUBROUTINE FILET (CP,CQ,CR,CS,XP,XQ,XR,XS,DUSE,CON,
      1                 THIC,ANS,N,L,FIX,SPACE,NREP)
 90    FORMAT (1H , 10E12.5)
 100   FORMAT(1H1/24H  TEMP VS TIME  AT DEPTH , F7.2,4H CM. /1H0 )
 101   FORMAT(1H0,80X,28HTIME IN HOURS FROM SUNRISE  /1H0,
      1        16H      TIME STEPS   ,F7.2,6H HOURS  )
 22    FORMAT(1H1)
       DIMENSION N(25),CP(25),CQ(25),CR(25),CS(25),DUSE(5),L(5)
       DIMENSION ANS(999),XP(25),XQ(25),XR(25),XS(25),CON(5),THIC(5)
       DIMENSION  TIME(999), ARRAY(800)
       NREP=NREP
       DO 1 I=1,NREP,1
       ANS(I)=0.0
 1     CONTINUE
       K=0
 2     M=0
       K=K+1
       IF (N(K)) 9,8,8
 8     X=FIX
       TERM=N(K)
 3     M=M+1
       XCAR=X
       IF (L(M)) 4,5,6
 4     CALL ERRORQ (5,2)
 5     IF (X-THIC(M)) 6,7,7
 7     XCAR=THIC(M)
 6     CALL CROSS (XP(K),XQ(K),XR(K),XS(K),CP(K),CQ(K),CR(K),CS(K),
      1              DUSE(M),CON(M),EGA,ZETA,TERM)
       CALL CARRY (XP(K),XQ(K),XR(K),XS(K),CP(K),CQ(K),CR(K),CS(K),
      1              CON(M),EGA,ZETA,XCAR)
       IF (L(M)) 4,13,11
 13    IF (X-THIC(M)) 11,11,10
 10    X=X-THIC(M)
       GO TO 3
 11    TAU=0.0
       DO 12 I=1,NREP,1
       ANS(I)=ANS(I)+CP(K)*COSF(EGA*TAU)+CQ(K)*SINF(EGA*TAU)
       TIME(I)=TAU
       TAU=TAU+SPACE
 12    CONTINUE
       GO TO 2
 9     DO 14 I=1,NREP,10
       K=I+9
       WRITE OUTPUT TAPE 3,90, (ANS(J),J=I,K)
 14    CONTINUE
       DO 202 I=1,NREP
       IF(ANS(I)-100.0)21,201,201
 201   IF(ANS(I)-400.0)202,21,21
 202   CONTINUE
```

```
20    NVL=((NREP-1)/10)+1
      NSBV=100/(NVL+1)
      CALL PLOT1(0,5,10,NVL,NSBV)
      CALL PLOT2(ARRAY,TIME(NREP)+SPACE,0.0,400.0,100.0)
      CALL PLOT3(1H*,TIME,ANS,NREP)
      WRITE OUTPUT TAPE 3,100,FIX
      CALL FPLOT4(36,36H       TEMPERATURE   *DEGREES KELVIN*   )
      WRITE OUTPUT TAPE 3,101,SPACE
21    WRITE OUTPUT TAPE 3,22
      RETURN
      END
```

Entry Name	- FILEX
Category	- Subroutine - Phase 1 only.
Purpose	- To calculate and report as output a profile of normal temperatures vs. depth for fixed time.
Arguments	- CP, CQ, CR, CS, XP, XQ, XR, XS, DUSE, CON, THIC, ANS, N, L, FIX, SPACE, NREP

L : The vector L(M) which indicates type of definition for thickness of each layer M,

FIX : The fixed time for the profile,

SPACE : Depth between successive values in the profile,

NREP : Number of values in the profile beginning with value at x = 0.

All other arguments appear in the glossary.

Error Exits	- None
Description	- In the original concept, it seemed likely that this subroutine would be used. The necessary coding was postponed until needed. No need arose. Hence, the present version simply prints an appropriate message and returns.
Lower Memory Requirements	- 30
Transfer Vector	- (STH), (FIL)
Common Requirements	- None

```
        LIST
        LABEL
        SUBROUTINE FILEX (CP,CQ,CR,CS,XP,XQ,XR,XS,DUSE,CON,
       1                  THIC,ANS,N,L,FIX,SPACE,NREP)
   90   FORMAT (1H ,31HX PROFILE SUBRTN. IS INCOMPLETE)
        DIMENSION N(25),CP(25),CQ(25),CR(25),CS(25),DUSE(5),L(5)
        DIMENSION ANS(999),XP(25),XQ(25),XR(25),XS(25),CON(5),THIC(5)
        WRITE OUTPUT TAPE 3,90
        RETURN
        END
```

Entry Name	- CROSS
Category	- Subroutine - Phase 1 only.
Purpose	- To apply equation A4 for a single value of $\omega \neq 0$ (or A2a for $\omega = 0$).
Arguments	- XP, XQ, XR, XS, CP, CQ, CR, CS, DUSE, CON, EGA, ZETA, TERM TERM : N(K) (floating point) to be used in calculating EGA. All other arguments appear in the glossary.
Error Exits	- #5 TERM is negative
Description	- In addition to P, Q, R, and S, EGA and ZETA are also calculated.
Lower Memory Requirements	- 122
Transfer Vector	- SQRT, ERRORQ
Common Requirements	- None

RE-ORDER No 61-422


```
*       LIST
*       LABEL
CLUNAR1    ALPHABET FOR LABEL
        SUBROUTINE CROSS (XP,XQ,XR,XS,CP,CQ,CR,CS,DUSE,CON,EGA,ZETA,TERM)
        EGA=0.0088656*TERM
        ZETA=SQRTF(0.5*EGA/DUSE)
        IF (TERM) 1,2,3
 1      CALL ERRORQ (5,2)
 2      XP=CP
        XQ=CQ
        XR=-CR/CON
        GO TO 4
 3      SP=0.5*CP
        SQ=0.5*CQ
        USE=-4.0*ZETA*CON
        SR=CR/USE
        SS=CS/USE
        XP=SP+SR-SS
        XQ=SQ+SR+SS
        XR=SP-SR+SS
        XS=SQ-SR-SS
 4      CONTINUE
        RETURN
        END
```

Entry Name	- CARRY
Category	- Subroutine - Phase 1 only.
Purpose	- To apply equation A6 and A7 for a single value of $\omega \neq 0$ (or A2a if $\omega = 0$).
Arguments	- XP, XQ, XR, XS, CP, CQ, CR, CS, CON, EGA, ZETA, X. All other arguments appear in the glossary. X : Depth at which A7 is to be evaluated (or A2a if $\omega = 0$)
Error Exits	- #5 EGA is negative
Lower Memory Requirements	- 153
Transfer Vector	- ERRORQ, EXP, COS, SIN
Common Requirements	- None

RE-ORDER No. 61-422


```
*       LIST
*       LABEL
CLUNAR1     ALPHABET FOR LABEL
        SUBROUTINE CARRY (XP,XQ,XR,XS,CP,CQ,CR,CS,CON,EGA,ZETA,X)
        IF (EGA) 1,2,3
 1      CALL ERRORQ (5,2)
 2      CP=XP+XR*X
        CQ=XQ
        GO TO 4
 3      ZX=ZETA*X
        ER=EXPF(ZX)
        CE=COSF(ZX)
        SE=SINF(ZX)
        SP=(XP*CE+XQ*SE)*ER
        SQ=(XQ*CE-XP*SE)*ER
        SR=(XR*CE-XS*SE)/ER
        SS=(XR*SE+XS*CE)/ER
        CP=SP+SR
        CQ=SQ+SS
        CR=-CON*ZETA*(SP-SR+SQ-SS)
        CS=CON*ZETA*(SP-SR-SQ+SS)
 4      CONTINUE
        RETURN
        END
```

Entry Name - ERRORQ and DMPCOR

Category - Subroutines - Both Phase 1 and Phase 2.

Purpose - To report the nature of and dump core due to a serious error.

Arguments - ITYPE, IDUMP for ERRORQ.

ITYPE : Error identification number,
IDUMP : Signal for DUMP,
= 1 return after printing ITYPE,
≠ 1 call DMPCOR and DUMP.
DMPCOR has no arguments.

Description - DMPCOR dumps all of core with mnemonics.

Lower Memory Requirements - 57 and 11

Transfer Vector - (SPH), (FIL), (STH), DMPCOR for ERRORQ
DUMP for DMPCOR

Common Requirements - None

RE-ORDER No. 61-422


```
*       LIST
*       LABEL
        SUBROUTINE ERRORQ (ITYPE,IDUMP)
 92     FORMAT (1H1,10HERROR NO. ,I5)
 93     FORMAT (1H ,12HDUMP FOLLOWS)
        PRINT 92,ITYPE
        WRITE OUTPUT TAPE 3,92,ITYPE
        IF (IDUMP-1) 2,1,2
 2      PRINT 93
        WRITE OUTPUT TAPE 3,93
        CALL DMPCOR
 1      CONTINUE
        RETURN
        END
```

```
*       LIST
*       LABEL
*       FAP
        COUNT   2
*ERRORQ   ALPHABET TO LABEL DMPCOR FOR INCLUSION IN BINARY DECK ERRORQ
        ENTRY   DMPCOR
DMPCOR  CALL    DUMP,00,9999,3,10000,19999,3,20000,32767,3
        END
```

RE-ORDER No. 61-422

Entry Name	- LUNAR2
Category	- Main Program - Phase 2
Purpose	- Compute temperature perturbations due to the disc and direct output as specified by control cards.
Arguments	- None (main program)
Dimension and Common Considerations	- Standard Phase 2 COMMON and DIMENSION statements are used. In addition, S(J,K) is set S(254,103).
Error Exits	- #98 - specified problem will not fit in the dimension restriction of S. #99 - 7090 failure.
Unusual Cautions	- The arrangement of data within the matrix S is dependent on the input parameters. Restrictions on input are considered on Page E42.
Description	- Calculation is made as described in Appendix B after input parameters have been read. Output is via subroutine LUAU2.
Lower Memory Requirements	- 943
Transfer Vector	- (FPT), FOURCO, LAYERS, (TSH), (RTN), (STH), (FIL), LUAU2, EXIT, VALUE, RADIO, BIQUAD, ERRORQ.
Common Requirements	- Standard LUNAR2 26,618.

RE-ORDER No. 61-422


```
*       LIST 8
*       LABEL
CLUNAR2     ALPHABET FOR LABEL
        COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
        COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
        COMMON NDELT,NDP
        COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
        DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
        DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
        DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
        DIMENSION S(254,103)
 85     FORMAT (1H0,15HINITIAL TIME = ,1E12.4)
 89     FORMAT (6E12.4)
 88     FORMAT (6I7)
 92     FORMAT (1X,6HDELT= ,E10.4,8H  DELX= ,E10.4,8H  DELR= ,E10.4)
 93     FORMAT (1X,6HNDELT=,I5,8H  NXANS=,I5,8H  NRANS=,I5,8H  NDISC=,I5,
       1      8H  NFDS=,I5,7H  NSDS=,I5,10H  HEIGHT= ,E10.4)
        CALL FOURCO (2,1)
        CALL LAYERS (1)
        READ INPUT TAPE 2,89,ESH,TSH,VIS,Q,ESU,SIG
        READ INPUT TAPE 2,89,DELT,DELX,DELR,TIN,HEIGHT
        READ INPUT TAPE 2,88,NDELT,NXANS,NRANS,NDISC,NFDS,NSDS
        WRITE OUTPUT TAPE 3,92,DELT,DELX,DELR
        WRITE OUTPUT TAPE 3,85,TIN
        WRITE OUTPUT TAPE 3,93,NDELT,NXANS,NRANS,NDISC,NFDS,NSDS,HEIGHT
        DO 18 I=1,NLAYER,1
        NDP(I)=THIC(I)/DELX+0.5
 18     CONTINUE
        NLAYER=NLAYER
        NDP(NLAYER)=32000
        A=DELX*ESU*SIG/CON(1)
        A=(2.0*A)/3.0
        FIX=0.0
        DXS=DELX*DELX
        DRS=DELR*DELR
        TSXS=DELT/DXS
        TSRS=DELT/DRS
        NDX=1
        MS=NSDS-1
        JR=NDISC-2
        IF (NDELT-NXANS) 31,31,32
 31     NTA=NDELT
        NTB=0
        GO TO 76
 32     NTA=(NXANS+NDELT)/2
        NTB=(NDELT-NXANS+1)/2
 76     JINK=1
        IF (NDISC) 75,75,33
 75     JINK=-1
        JR=NRANS+NDELT+1
        NDISC=10000
        IF (JR-NFDS) 37,98,98
 33     JR=JR+1
```

```
      IF (JR+1+NTA-NTB-NRANS) 33,36,36
36    IF (JR+1+NTA-NFDS) 37,98,98
37    IF (NTA+2-NSDS) 38,38,98
38    CONTINUE
      DO 1 J=1,NFDS,1
      DO 39 K=1,NSDS,1
      S(J,K)=0.0
39    CONTINUE
1     CONTINUE
      MB=MS+2
      NTIM=0
      T=0.0
      DO 24 NT=1,NTA,1
      JR=JR+JINK
      T=T+DELT
      NTIM=NTIM+1
      TIME=TIN+T
      MB=MB-1
      IF (MS-MB) 17,22,22
22    GO TO 2
2     LAY=1
      NEDP=NDP(1)
      LEX=0
      DO 9 MX=MB,MS,1
      M=MX
      LEX=LEX+1
      IF (LEX-NEDP) 5,3,6
3     IF (MS-MX) 99,7,4
4     LAY=LAY+1
      M=M+1
5     MSTO=M-1
      GO TO 10
6     LAY=LAY-1
      M=M-1
7     MSTO=M-1
      GO TO 12
8     LAY=LAY+1
      NEDP=NEDP+NDP(LAY)
9     CONTINUE
17    M=MB-1
      MSTO=M-1
      GO TO 14
24    CONTINUE
      IF (NTB) 99,34,35
35    NDX=2
      MU=NTA
      MLAY=LAY
      NEDP=NEDP-NDP(LAY)
      DO 25 NT=1,NTB,1
      JR=JR-1
      T=T+DELT
      NTIM=NTIM+1
      TIME=TIN+T
```

```
      NU=MU
      MU=MU-1
      LAY=MLAY
      MB=NSDS-1
      MEDP=NEDP
      DO 20 MX=1,MU,1
      MB=MB-1
      M=MB
      MSTO=M+1
      NU=NU-1
      IF (NU-MEDP) 28,26,10
26    IF (MX-1) 99,29,27
27    LAY=LAY-1
      M=M-1
      MSTO=MSTO-1
      GO TO 10
28    M=M+3
      MSTO=MSTO+1
      GO TO 12
29    LAY=LAY-1
      MLAY=LAY
      NEDP=NEDP-NDP(LAY)
      DO 30 J=1,JR,1
      S(J,MSTO)=0.0
30    CONTINUE
23    MEDP=MEDP-NDP(LAY)
20    CONTINUE
      M=MSTO
      MSTO=MSTO-1
      GO TO 14
25    CONTINUE
34    CONTINUE
      CALL LUAU2 (NTIM,MSTO,JR)
      CALL EXIT
16    CONTINUE
      CALL LUAU2 (NTIM,MSTO,JR)
      GO TO (24,25),NDX
10    CNR=0.0
      DO 11 J=2,JR,1
      CNR=CNR+2.0
      ANS=S(J,M)+DUSE(LAY)*((S(J,M-1)+S(J,M+1)-2.0*S(J,M))*TSXS
     1   +((S(J+1,M)-S(J-1,M))/CNR+S(J+1,M)+S(J-1,M)-2.0*S(J,M))*TSRS)
      S(J,MSTO)=ANS
11    CONTINUE
      S(1,MSTO)=S(2,MSTO)
      GO TO (9,20),NDX
12    DO 13 J=2,JR,1
      ANS=(CON(LAY)*S(J,M-2)+CON(LAY+1)*S(J,M))/(CON(LAY)+CON(LAY+1))
      S(J,MSTO)=ANS
13    CONTINUE
      S(1,MSTO)=S(2,MSTO)
      GO TO (8,23),NDX
```

```
14    CALL VALUE (0.0,TIME,V,F)
      R=0.0
      DO 15 J=2,JR,1
      R=R+DELR
      RAD=HEIGHT
      CALL RADIO (R,TIME,RAD)
      B=V + ( (4.0*S(J,M)-S(J,M+1)+2.0*DELX*(RAD+F))/3.0 )
      CALL BIQUAD(A,B,400.0,0.001,ANS)
      S(J,MSTO)=ANS-V
15    CONTINUE
      S(1,MSTO)=S(2,MSTO)
      GO TO 16
99    CALL ERRORQ (99,2)
98    CALL ERRORQ (98,2)
      END
```

RE-ORDER No. 61-422

Entry Name	-	LUNE2M
Category	-	Main Program - Phase 2.
Purpose	-	Compute temperature perturbations due to the disc and direct output as specified by control cards.
Arguments	-	None (main program)
Dimension and Common Considerations	-	Standard Phase 2 COMMON and DIMENSION statements are used. In addition, S(K) is set S(25000).
Error Exits	-	#98 - specified problem will not fit in the dimension restriction of S. #99 - 7090 failure.
Unusual Cautions	-	The size of S(K) needed is dependent on the input parameters. Restrictions on input are considered on Page E42.
Description	-	Calculation is made as described in Appendix B after input parameters have been read assuming that the independent variable, r(radial distance) is irrelevant. Output is via subroutine LUNO2.
Lower Memory Requirements	-	775
Transfer Vector	-	(FPT), FOURCO, LAYERS, (TSH), (RTN), (STH), (FIL), LUNO2, VALUE, RADIO, BIQUAD, ERRORQ.
Common Requirements	-	Standard LUNE2M 25,456.

RE-ORDER No. 61-422


```
*       LIST 8
*       LABEL
CLUNE2M      LUNAR2 MODIFIED FOR ONE-D. PROBLEM
        COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
        COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
        COMMON NDELT,NDP
        COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
        DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
        DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
        DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
        DIMENSION S(25000)
 85     FORMAT (1H0,15HINITIAL TIME = ,1E12.4)
 89     FORMAT (6E12.4)
 88     FORMAT (6I7)
 92     FORMAT (1X,6HDELT= ,E10.4,8H  DELX= ,E10.4,8H  DELR= ,E10.4)
 93     FORMAT (1X,6HNDELT=,I5,8H  NXANS=,I5,8H  NRANS=,I5,8H  NDISC=,I5,
      1       8H  NFDS=,I5,7H  NSDS=,I5,10H  HEIGHT= ,E10.4)
 100    FORMAT (1X,1A6)
        CALL FOURCO (2,1)
 102    CALL LAYERS (1)
        READ INPUT TAPE 2,89,ESH,TSH,VIS,Q,ESU,SIG
        READ INPUT TAPE 2,89,DELT,DELX,DELR,TIN,HEIGHT
        READ INPUT TAPE 2,88,NDELT,NXANS,NRANS,NDISC,NFDS,NSDS
        WRITE OUTPUT TAPE 3,92,DELT,DELX,DELR
        WRITE OUTPUT TAPE 3,85,TIN
        WRITE OUTPUT TAPE 3,93,NDELT,NXANS,NRANS,NDISC,NFDS,NSDS,HEIGHT
        DO 18 I=1,NLAYER,1
        NDP(I)=THIC(I)/DELX+0.5
 18     CONTINUE
        NLAYER=NLAYER
        NDP(NLAYER)=32000
        A=DELX*ESU*SIG/CON(1)
        A=(2.0*A)/3.0
        FIX=0.0
        DXS=DELX*DELX
        TSXS=DELT/DXS
        NDX=1
        NSDS=25000
        MS=NSDS-1
        NDISC=10000
        JR=2
        IF (NDELT-NXANS) 31,31,32
 31     NTA=NDELT
        NTB=0
        GO TO 37
```

1:794.38-E23

```
32    NTA=(NXANS+NDELT)/2
      NTB=(NDELT-NXANS+1)/2
37    IF (NTA+2-NSDS) 38,38,98
38    CONTINUE
      DO 39 K=1,NSDS,1
      S(K)=0.0
39    CONTINUE
      MB=MS+2
      NTIM=0
      T=0.0
      DO 24 NT=1,NTA,1
      T=T+DELT
      NTIM=NTIM+1
      TIME=TIN+T
      MB=MB-1
      IF (MS-MB) 17,22,22
22    GO TO 2
2     LAY=1
      NEDP=NDP(1)
      LEX=0
      DO 9 MX=MB,MS,1
      M=MX
      LEX=LEX+1
      IF (LEX-NEDP) 5,3,6
3     IF (MS-MX) 99,7,4
4     LAY=LAY+1
      M=M+1
5     MSTO=M-1
      GO TO 10
6     LAY=LAY-1
      M=M-1
7     MSTO=M-1
      GO TO 12
8     LAY=LAY+1
      NEDP=NEDP+NDP(LAY)
9     CONTINUE
17    M=MB-1
      MSTO=M-1
      GO TO 14
24    CONTINUE
      IF (NTB) 99,34,35
35    NDX=2
      MU=NTA
      MLAY=LAY
      NEDP=NEDP-NDP(LAY)
      DO 25 NT=1,NTB,1
      T=T+DELT
      NTIM=NTIM+1
      TIME=TIN+T
      NU=MU
      MU=MU-1
      LAY=MLAY
      MB=NSDS-1
```

```
      MEDP=NEDP
      DO 20 MX=1,MU,1
      MB=MB-1
      M=MB
      MSTO=M+1
      NU=NU-1
      IF (NU-MEDP) 28,26,10
26    IF (MX-1) 99,29,27
27    LAY=LAY-1
      M=M-1
      MSTO=MSTO-1
      GO TO 10
28    M=M+3
      MSTO=MSTO+1
      GO TO 12
29    LAY=LAY-1
      MLAY=LAY
      NEDP=NEDP-NDP(LAY)
      S(MSTO)=0.0
23    MEDP=MEDP-NDP(LAY)
20    CONTINUE
      M=MSTO
      MSTO=MSTO-1
      GO TO 14
25    CONTINUE
34    CONTINUE
      CALL LUN02 (NTIM,MSTO,JR)
101   READ INPUT TAPE 2,100,TEST
      IF (TEST-6HREPEAT) 101,102,101
16    CONTINUE
      CALL LUN02 (NTIM,MSTO,JR)
      GO TO (24,25),NDX
10    CONTINUE
      ANS=S(  M)+DUSE(LAY)*((S(  M-1)+S(  M+1)-2.0*S(  M))*TSXS )
      S(MSTO)=ANS
      GO TO (9,20),NDX
12    CONTINUE
      ANS=(CON(LAY)*S(  M-2)+CON(LAY+1)*S(  M))/(CON(LAY)+CON(LAY+1))
      S(MSTO)=ANS
      GO TO (8,23),NDX
14    CALL VALUE (0.0,TIME,V,F)
      R=0.0
      RAD=HEIGHT
      CALL RADIO (R,TIME,RAD)
      B=V + ( (4.0*S(  M)-S(  M+1)+2.0*DELX*(RAD+F))/3.0 )
      CALL BIQUAD(A,B,400.0,0.001,ANS)
      S(  MSTO)=ANS-V
      GO TO 16
99    CALL ERRORQ (99,2)
98    CALL ERRORQ (98,2)
      END
```

Entry Name - LUAU2

Category - Subroutine - Phase 2.

Purpose - Output Subroutine for Phase 2.

Arguments - NTIM, MSTO, JR

NTIM : Number of time steps for which the Phase 2 calculation has been completed,

MSTO : Value of the column (last dimension of S) in which surface temperature perturbations currently appear,

JR : Number of r-steps to last non-zero perturbation.

Dimension and Common Considerations - Standard Phase 2 COMMON and DIMENSION statements are used. In addition, S(J,K) is set S(254,103).

Error Exits - None

Unusual Cautions - The dimensionality of S must agree with that given in the main program, LUNAR2.

Description - Output consists of radial profiles for fixed depth as specified by control cards read by LUAU2. If NTIM is sufficiently small, no output occurs.

Lower Memory Requirements - 223

Transfer Vector - (TSH), (RTN), (STH), (FIL)

RE-ORDER No. 61-422


```
*       LIST
*       LABEL
        SUBROUTINE LUAU2 (NTIM,MSTO,JR)
        COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
        COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
        COMMON NDELT,NDP
        COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
        DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
        DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
        DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
        DIMENSION S(254,103)
        DIMENSION ANS(10)
86      FORMAT (1H0,19HRADIAL PROFILE. T= ,E14.8,5H  X= ,E14.8,
      1         11H  R-SPACE= ,E14.8)
87      FORMAT (1E12.4,3I7)
90      FORMAT (1H ,10E11.4)
        IF (FIX) 2,1,2
1       READ INPUT TAPE 2,87,FIX,MOX,MDRS,NDRS
        NFIT=FIX/DELT
2       IF (NFIT-NTIM) 1,3,4
4       RETURN
3       LOX=MSTO+MOX
        WORKA=MOX
        WORKA=WORKA*DELX
        WORKB=MDRS
        WORKB=WORKB*DELR
        WRITE OUTPUT TAPE 3,86,FIX,WORKA,WORKB
        K=1
10      DO 5 I=1,10,1
        ANS(I)=0.0
5       CONTINUE
        DO 9 J=1,10,1
        IF (NSDS-1-LOX) 6,7,7
7       IF (JR-K) 6,8,8
8       ANS(J)=S(K,LOX)
6       K=K+MDRS
9       CONTINUE
        WRITE OUTPUT TAPE 3,90,(ANS(I),I=1,10)
        NDRS=NDRS-10
        IF (NDRS) 1,1,10
        END
```

Entry Name	- LUNO2
Category	- Subroutine - Phase 2
Purpose	- Output subroutine for Phase 2 modified for one-dimensional problem.
Arguments	- NTIM, MSTO, JR NTIM : Number of time steps for which the Phase 2 calculation has been completed, MSTO : Value of the column (only dimension of S) in which surface temperature perturbations currently appear, JR : Meaningless (LUNO2 is a modification of LUAU2).
Dimension and Common Considerations	- Standard Phase 2 COMMON and DIMENSION statements are used. In addition, S(K) is set S(25000).
Error Exits	- None
Unusual Cautions	- The dimensionality of S <u>must</u> agree with that given in the main program, LUNE2M.
Description	- Output consists of depth profiles for fixed time as specified by control cards read by LUNO2. If NTIM is sufficiently small, no output occurs.
Lower Memory Requirements	- 188
Transfer Vector	- (TSH), (RTN), (STH), (FIL)

```
*       LIST
●       LABEL
        SUBROUTINE LUN02 (NTIM,MSTO,JR)
        COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
        COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
        COMMON NDELT,NDP
        COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
        DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
        DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
        DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
        DIMENSION S(25000)
        DIMENSION ANS(10)
 86     FORMAT (1H0,18HDEPTH PROFILE. T= ,E14.8,11H  FIRST-X= ,E14.8,
       1        11H  X-SPACE= ,E14.8)
 87     FORMAT (1E12.4,3I7)
 90     FORMAT (1H ,10E11.4)
        IF (FIX) 2,1,2
 1      READ INPUT TAPE 2,87,FIX,MOX,MDRS,NDRS
        NFIT=FIX/DELT
 2      IF (NFIT-NTIM) 1,3,4
 4      RETURN
 3      LOX=MSTO+MOX
        WORKA=MOX
        WORKA=WORKA*DELX
        WORKB=MDRS
        WORKB=WORKB*DELX
        WRITE OUTPUT TAPE 3,86,FIX,WORKA,WORKB
 10     DO 5 I=1,10,1
        ANS(I)=0.0
 5      CONTINUE
        DO 9 J=1,10,1
        IF (NSDS-1-LOX) 8,7,7
 7      ANS(J)=S(LOX)
 8      LOX=LOX+MDRS
 9      CONTINUE
        WRITE OUTPUT TAPE 3,90,(ANS(I),I=1,10)
        NDRS=NDRS-10
        IF (NDRS) 1,1,10
        END
```

Entry Name	- RADIO
Category	- Subroutine - Phase 2
Purpose	- Computes surface radiation input divided by conductivity of surface layer.
Arguments	- R, T, RAD R : radius (cm. from center of disc), T : time (hours from nearest previous sunrise), RAD : input argument - height of disc above surface; output - surface radiation/CON(1).
Dimension and Common Considerations	- Standard Phase 2 COMMON and DIMENSION statements are used.
Error Exits	- None
Unusual Cautions	- T must be positive and less than one lunar cycle.
Description	- The angular effect of the geometry of the disc is calculated if height is positive, otherwise this effect is ignored.
Lower Memory Requirements	- 179
Transfer Vector	- SIN, SQRT

```
●       LIST
*       LABEL
CRADIOX  RADIATION INPUT FOR LUNAR2 WITH RADIATION EDGE EFFECTS
        SUBROUTINE RADIO(R,T,RAD)
        COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
        COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
        COMMON NDELT,NDP
        COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
        DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
        DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
        DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
        HEIGHT=RAD
        ANDISC=NDISC
        RSH=ANDISC*DELR
        IF(HEIGHT)8,1,7
 1      RAD=0.0
        IF(RSH-R)3,2,2
 2      RAD=ESH*ESU*SIG*TSH**4/CON(1)
        GO TO 8
 3      IF(EGA(2)*T-3.1415927)4,5,5
 4      SOL=VIS*Q*SINF(EGA(2)*T)/CON(1)
        GO TO 6
 5      SOL=0.0
 6      RAD=RAD+SOL
        GO TO 8
 7      B=RSH/HEIGHT
        C=R/HEIGHT
        F=0.5(1.0-(1.0 + C**2 - B**2)/SQRTF(C**4 + 2.0*C**2 *(1.0-B**2)
      1   +(1.0 + B**2)**2))
        RAD=ESH*ESU*SIG*F*TSH**4/CON(1)
        IF(RSH-R)3,5,5
 8      RETURN
        END
```

RE-ORDER No. 61-422

Entry Name	- LAYERS
Category	- Subroutine - Phase 2.
Purpose	- To read description of layers.
Arguments	- IOUT IOUT : Specifies output as follows: $\geq$ 1 output of layer descriptions, $<$ 1 no output layer descriptions.
Dimension and Common Considerations	- Standard Phase 2 COMMON and DIMENSION Statements are used.
Error Exits	- None
Unusual Cautions	- L(M) is the fixed point variable for layer M which indicates: L(M) = 0 : layer thickness is defined, L(M) > 0 : layer thickness is undefined (∞), L(M) < 0 : Call EXIT immediately. In addition to reading L(M), CON(M), THIC(M), RHO(M), and SPEH(M), LAYERS computes DUSE(M) = CON(M)/(RHO(M)* SPEH(M)). The infinite layer is, of course, read last and serves as a signal that all layering descriptions have then been read. LAYERS also sets NLAYER.
Lower Memory Requirements	- 138
Transfer Vector	- (STH), (FIL), (TSH), (RTN), EXIT.

RE-ORDER No. 61-422


```
*       LIST
*       LABEL
        SUBROUTINE LAYERS (IOUT)
 90     FORMAT (1X,1I3,4E14.8)
 94     FORMAT (1H0,18HPHYSICAL CONSTANTS)
 95     FORMAT (1H ,1I3,4E14.8)
        COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
        COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
        COMMON NDELT,NDP
        COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
        DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
        DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
        DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
        K=0
        IF (IOUT) 2,2,1
 1      WRITE OUTPUT TAPE 3,94
 2      K=K+1
        READ INPUT TAPE 2,90,L,CON(K),THIC(K),RHO(K),SPEH(K)
        IF (IOUT) 4,4,3
 3      WRITE OUTPUT TAPE 3,95,L,CON(K),THIC(K),RHO(K),SPEH(K)
 4      IF (L) 7,5,8
 8      THIC(K)=1.0E30
 5      DUSE(K)=CON(K)/(RHO(K)*SPEH(K))
        IF (L) 7,2,6
 6      NLAYER=K
        RETURN
 7      CALL EXIT
        END
```

RE-ORDER No. 61-422

Entry Name	- FOURCO
Category	- Subroutine - Phase 2.
Purpose	- Read Lunar Fourier coefficients and prepare these coefficients for further use.
Arguments	- IPROB, IOUT IPROB : specifies output title as follows: $\leq$ 1 title for Phase 1, = 2 title for Phase 2, > 2 no title. IOUT : specifies output of Fourier coefficients: $\leq$ 0 no output of coefficients, > 0 output of coefficients for reference.
Dimension and Common Considerations	- Standard Phase 2 COMMON and DIMENSION statements are used.
Error Exits	- None
Description	- In addition to reading of Fourier coefficients into DP(K), DQ(K), DR(K), and DS(K); FOURCO reads N(K), calculates EGA(K), rescales DR(K) and DS(K) for time units in hours, and sets NFORCO.
Lower Memory Requirements	- 150
Transfer Vector	- (STH), (FIL), (TSH), (RTN).

RE-ORDER No. 61-422


```
*       LIST
*       LABEL
        SUBROUTINE FOURCO (IPROB,IOUT)
 90     FORMAT (1X,1I3,4E14.8)
 91     FORMAT (1H0,20HFOURIER COEFFICIENTS)
 93     FORMAT (1H1,27HLUNAR1 TEMPERATURE PROFILES)
 95     FORMAT (1H ,1I3,4E14.8)
 98     FORMAT (1H1,32HLUNAR2 TEMPERATURE PERTURBATIONS)
        COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
        COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
        COMMON NDELT,NDP
        COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
        DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
        DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
        DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
        K=0
        IF (IPROB-2) 1,2,3
 1      WRITE OUTPUT TAPE 3,93
        GO TO 3
 2      WRITE OUTPUT TAPE 3,98
 3      IF (IOUT) 5,5,4
 4      WRITE OUTPUT TAPE 3,91
 5      K=K+1
        READ INPUT TAPE 2,90,N(K),DQ(K),DP(K),DS(K),DR(K)
        DS(K)=3600.0*DS(K)
        DR(K)=3600.0*DR(K)
        TERM=N(K)
        EGA(K)=0.0088656*TERM
        IF (IOUT) 7,7,6
 6      WRITE OUTPUT TAPE 3,95,N(K),DQ(K),DP(K),DS(K),DR(K)
 7      IF (N(K)) 8,5,5
 8      NFORCO=K-1
        RETURN
        END
```

Entry Name	- VALUE
Category	- Subroutine - Phase 2
Purpose	- To calculate temperature and x-derivative of temperature.
Arguments	- X, T, V, F. X : Depth at which results are desired. T : Time (measured from nearest previous sunrise) at which results are desired. V : Temperature. F : x-derivative of temperature.
Dimension and Common Considerations	- Standard Phase 2 COMMON and DIMENSION statements are used.
Error Exits	- #10 - the argument, X, is negative. Also, the argument, X, is positive.
Unusual Cautions	- In the original concept, it seemed likely that this subroutine would be used for positive X. The necessary coding was postponed until needed. No need arose.
Description	- Using the Fourier coefficients in COMMON, Phase 1 V and F are calculated.
Lower Memory Requirements	- 88
Transfer Vector	- ERRORQ, COS, SIN

RE-ORDER No. 61-422


```
●      LIST
*      LABEL
       SUBROUTINE VALUE (X,T,V,F)
       COMMON N,DP,DQ,DR,DS,CP,CQ,CR,CS,XP,XQ,XR,XS,EGA,ZETA,NFORCO,CON
       COMMON THIC,RHO,SPEH,DUSE,NLAYER,FIX,SPACE,DELT,DELX,DELR,TSXS
       COMMON NDELT,NDP
       COMMON TSRS,NFDS,NSDS,NXANS,NRANS,NDISC,ESH,TSH,VIS,Q,ESU,SIG,S
       DIMENSION N(25),DP(25),DQ(25),DR(25),DS(25),EGA(25),ZETA(25)
       DIMENSION CP(25),CQ(25),CR(25),CS(25),XP(25),XQ(25),XR(25),XS(25)
       DIMENSION CON(10),THIC(10),RHO(10),SPEH(10),DUSE(10),NDP(10)
       IF (X) 3,4,5
 5     CONTINUE
 3     CALL ERRORQ (10,2)
 4     V=0.0
       F=0.0
       DO 1 J=1,NFORCO,1
       TAU=T*EGA(J)
       CUS=COSF(TAU)
       SAN=SINF(TAU)
       V=V+DP(J)*CUS+DQ(J)*SAN
       F=F+DR(J)*CUS+DS(J)*SAN
 1     CONTINUE
 2     F=-F/CON(1)
       RETURN
       END
```

Entry Name	- BIQUAD
Category	- Subroutine - Phase 2.
Purpose	- To find the positive root of $g(U) = AU^4 + U - B$ where $A, B > 0$.
Arguments	- A, B, UMAX, FRACT, ROOT A,B : Coefficients of g(0). UMAX : Maximum allowable root (°K, ∴ positive); a root is assumed to lie in (0, UMAX). FRACT: A root is supposed found if known to lie in a U-interval of length not exceeding FRACT. ROOT : The desired positive root.
Error Exits	- #6 - no root discovered in (0, UMAX).
Unusual Cautions	- If A and B are such that the root would lie outside (0, UMAX), strange results occur. We successfully used, UMAX = 400.0.
Description	- The root search uses an alternating half interval - secant intersection method.
Lower Memory Requirements	- 141
Transfer Vector	- ERRORQ
Common Requirements	- None

```
*       LIST
*       LABEL
        SUBROUTINE BIQUAD (A, B, UMAX, FRACT, ROOT)
C       SUBROUTINE BIQUAD, ROOT OF BIQUADRATIC EQUATION, LUNAR2
        QUARTF(X)=(A*X**4)+X-B
        X1=0.0
        X2=UMAX
        DO 7 INDEX=1, 100
        Y1=QUARTF(X1)
        Y2=QUARTF(X2)
        XPRIME=((X1*Y2)-(Y1*X2))/(Y2-Y1)
        YPRIME=QUARTF(XPRIME)
        IF(YPRIME) 2, 1, 1
  1     ROOT=XPRIME
        GO TO 8
  2     X1=XPRIME
        XPRIME=(X1+X2)/2.0
        YPRIME=QUARTF(XPRIME)
        IF(YPRIME) 3, 1, 5
  3     X1=XPRIME
        GO TO 4
  5     X2=XPRIME
  4     IF(X2-X1-FRACT) 6, 6, 7
  6     ROOT=(X1+X2)/2.0
        GO TO 8
  7     CONTINUE
        CALL ERRORQ (6,2)
  8     RETURN
        END
```

GLOSSARY OF NAMES

I. Data Read by Both Phases

N(K) : Multipliers of Ω in calculating ω_n.
DP(K) : Temperature cosine coefficients.
DQ(K) : Temperature sine coefficients.
DR(K) : Flux cosine coefficients.
DS(K) : Flux sine coefficients.
CON(M) : Layer thermal conductivities (cals/hr cm °K).
THIC(M) : Layer thicknesses (cm).
RHΦ(M) : Layer densities (gm/cm^3).
SPEH(M) : Layer specific heats (cal/gm °K).

II. Data Read by Phase 2 Only

ESH : Emissivity of disc (we used 0.9).
TSH : Temperature of disc (we used 300°K).
VIS : Average reflectivity, α_v (we used 0.875).
Q : Solar constant, q_o (we used 117.0 cal/cm^2 hr).
ESU : Emissivity of lunar surface, ϵ_s (we used 0.9).
SIG : Stefan-Boltzmann constant, σ (cal/cm^2 hr (°K)^4).
DELT : Δt = time step (cm).
DELX : Δx = depth step (cm).
DELR : Δr = radial step (cm).
TIN : Initial time measured from first sunrise prior to placement of disc (hr).
HEIGHT : Height of disc above surface (cm).
NDELT : Number of time steps to be taken in this run.
NXANS : Number of depth steps for which answers are desired after NDELT time steps.
NRANS : Number of radial steps for which answers are desired after NDELT time steps.
NDISC : Number of radial steps from center of disc to nearest grid point outside the radius of the disc.

NFDS : First dimension of S.

NSDS : Second dimension of S.

III. Variables Calculated by Both Phases

DUSE(M) : Layer thermal diffusivities.

IV. Work Areas - Phase 1

CP(K), CQ(K), CR(K), and CS(K): Storage for data Fourier coefficients and results related to Equations A5 and A6.

XP(K), XQ(K), XR(K), and XS(K): Results of Equation A4.

EGA : Storage for ω_n of current interest.

ZETA : Storage for ζ_n of current interest.

V. Work Areas - Phase 2

EGA(K) : Storage for all ω_n.

VI. Variables Calculated by Phase 2 Only

NFORCO : Number of values of ω_n (including $\omega_o = 0$).

NLAYER : Number of layers.

TSXS : $\Delta t/(\Delta x)^2$.

TSRS : $\Delta t/(\Delta r)^2$.

NDP(M) : Number of depth steps required for layer M. Defined to be 32,000 for an infinite layer.

S(J,K) : Storage area and work area for results in LUNAR2.

S(K) : Storage area and work area for results in LUNO2 (the one-dimensional modification of LUNAR2).

INPUT SEQUENCE AND RESTRICTIONS

Phase 1:

1. Fourier Coefficients.

```
90      FORMAT (1X, 1I3, 4E14.8)
        READ INPUT TAPE 2, 90, N(K), DQ(K), DP(K), DS(K),
        DR(K)
```

Iterate: K = 0,1,2, ..., M - 1, M

with N(K) = 0,1,2, ..., M - 1, -1

N(M) = -01 signals termination of read.

DQ(M), DP(M), DS(M), DR(M) are arbitrary.

2. Layer Definition.

```
        READ INPUT TAPE 2, 90, L(K), CON(K), THIC(K),
        RHO(K), SPEH(K)
```

Iterate: K = 0,1,2, ..., M - 1, M

L(K) = 0,0,0, ..., 0, 1

L(M) = 1 signals termination of read.

Layer M is infinite in thickness.

L(M) = -1 signals immediate exit.

3. Profile Specification.

```
92      FORMAT (1X, 1I3, 2E14.8, 1I4)
        READ INPUT TAPE 2, 92, KFIL, FIX, SPACE, NREP
```

KFIL > 0 → Depth profile is desired,

KFIL = 0 → Time profile is desired,

KFIL < 0 → Return to 2 for new media.

See pages E4 and E9 for further information.

Phase 2 (LUNAR2: S(J,K)):

1. Fourier Coefficients - Precisely as in Phase 1.
2. Layer Definition - Precisely as in Phase 1.
3. Parameters and Specifications.

```
89      FORMAT (6E12.4)
88      FORMAT (6I7)
        READ INPUT TAPE 2, 89, ESH, TSH, VIS, Q, ESU, SIG
        READ INPUT TAPE 2, 89, DELT, DELX, DELR, TIN, HEIGHT
        READ INPUT TAPE 2, 88, NDELT, NXANS, NRANS, NDISC,
        NFDS, NSDS
```

Restrictions.

a) DELR = DELX and DELT satisfy Eqn. B7 for the largest diffusivity in the problem. THIC(K)/DELX an integer for each K with finite THIC(K).

b) NFDS and NSDS are the first and second numbers in the DIMENSION statement defining S(J,K).

c) Denote "greatest integer $\leq \alpha$" by $[\alpha]$, and define

$$NTA = MIN ([(NXANS + NDELT)/2], NDELT),$$

$$NTB = MAX ([(NDELT-NXANS + 1)/2], 0),$$

$$JR = MAX (NDISC-1, NRANS-NTA-1 + NTB).$$

Then it must be true that

$$JR + 1 + NTA < NFDS, \quad \text{and } 2 + NTA \leq NSDS$$

where NDISC = [(15.24 cm/DELR) + 0.99].

If these conditions cannot be satisfied, it will be necessary to re-dimension S.

It should be noted that LUNAR2 is specialized if the NDISC definition above is violated to the extent that NIDSC is not positive. This results in an inefficient use of LUNAR2 to solve the one-dimensional problem. This option should not be used since LUNE2M does the same work in less time.

4. Radial Profile Specification (Read by LUAU2, P. E26).

```
87      FORMAT (1E12.4, 3I7)
        READ INPUT TAPE 2, 87, FIX, MOX, MDRS, NDRS
```

FIX: Time at which a radial profile is desired. FIX should increase monotonically from profile to profile. FIX/DELT should be an integer.

MOX: Number of depth steps at which the profile is desired.

MDRS: Number of radial steps between values on the profile beginning with the r = o value.

NDRS: Number of values in the profile.

<u>Phase 2 (LUNE2M: S(K))</u>:

1. Fourier Coefficients - Precisely as above.
2. Layer Definition - Precisely as above.
3. Parameters and Specifications - Read Precisely as above.

Restrictions.

a) DELX and DELT satisfy Eqn. B7 for the largest diffusivity of the problem. THIC(K)/DELX an integer for each K with finite THIC(K). DELR, NRANS, NDISC, NFDS, and NSDS are arbitrary.

b) Denote "greatest integer $\leq \alpha$" by $[\alpha]$, and define.

$$NTA = MIN\ ([(NXANS + NDELT)/2],\ NDELT).$$

Then it must be true that

$$NTA + 2 \leq 25{,}000.$$

If not, the problem cannot be solved.

4. Depth Profile Specification (Read by LUNO2, P. E28).

```
87      FORMAT (1E12.4, 3I7)
        READ INPUT TAPE 2, 87, FIX, MOX, MDRS, NDRS
```

FIX: Time at which a depth profile is desired. FIX should increase monotonically from profile to profile. FIX/DELT should be an integer.

MOX: Number of depth steps to the first (nearest surface) value in the profile.

MDRS: Number of depth steps between successive profile values.

NDRS: Number of values in the profile.

5. Repetition Signal.

```
100     FORMAT (1X, 1A6)
        READ INPUT TAPE 2, 100, TEST
```

If the word REPEAT is read into TEST return is made to 2 above for a new media.

Restriction.

In order for the word REPEAT to be properly interpreted it is necessary that the list of depth profiles described above (4) contain at least one profile whose time (FIX) is greater than the time obtained by an NDELT summation of DELT.